

## Homework Module 4:

- (1) Use induction to show that the recurrence  $a_i = 2a_{i-1} + 1$  is solved by  $a_i = 2^i - 1$ .
- (2) Use induction to show that the recurrence  $a_i = 2/5a_{i-1} + 3/5a_{i-2}$ ,  $a_0 = 0, a_1 = 1$  is solved by  $a_i = -\frac{(-3)^i - 5^i}{8 \cdot 5^{i-1}}$ .
- (3) Use the substitution method to show that  $T(n) = T(n - 1) + n + 1$  implies that  $T(n) \leq Cn^2$  as long as  $C \geq 1$  and  $C \geq T(1)$ .
- (4) Draw a recursion tree for the recurrence  $T(n) = 3T(\lfloor n/2 \rfloor) + n$ . Derive a solution of the recurrence from the tree. For simplicity, you can assume that  $n$  is a power of two.