Final Preparation

Problem 1:

Create a 2-3 tree by inserting the following words:

ani, ant, boy, cat, cob, dab, dog, eel , elk, fly, eft, koi, gnu, fox, gib, hen, hog, pig, ox, kit, man, olm, pug, sow, ai, bee, boy,

Then delete ani, ant, and boy.

Problem 2:

What is the state of an LH-file with 20 buckets. Which buckets will records with key hashes 5, 10, 15, 20, 25, 30 contain?

Problem 3:

You are given a graph $G = (V, E)$ in the form of a list of adjacency lists [adj[v] for $v \in V$].

```
def color(G, s):
      for v in G.vertices:
            v.status = 0

      s.status = 1
      s.color = 'Blue'
      queue = []
      queue.append(s)

      while queue:
            u = queue.pop(0)
            for v in u.adjacency:
                  if v.color != None and v.color == u.color:
                        return False
                  if v.status == 0
                        v.status = 1
                        if u.color = 'Red':
                              v.color = 'Blue'
                        elif u.color = 'Blue':
                              v.color = 'Red'
                        queue.append(v)
      v.status = 2
      return True
```

This program finds a coloring of the edges in blue and red so that adjacent vertices are colored differently, or reports that this is impossible. What is the runtime of this algorithm in terms of $|V|$ and $|E|$.

Problem 4:

You are given an array of floating point values that is almost ordered. (For example, the array contains periodic measurements at an automatic sensor that for some weird reason sometimes delays writing a measurement for a little while.) You are guaranteed that array elements are at most 4 spots away from where they should be. The array has n elements. Give an algorithm with $O(n)$ runtime that will sort the elements. (Hint: Use a min-heap of size 4.)

Problem 5:

The Dutch Flag problem:

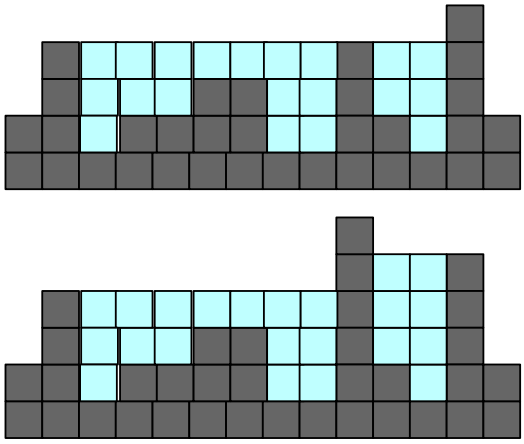
You are given an array of n values 'r', 'w', and 'b' that are not ordered. Give an algorithm that orders the array in time $O(n)$ so that it becomes the Dutch flag. You are only allowed constant memory in addition to storing the array.

Problem 6:

You are given a graph and are asked whether it is possible to color all vertices with k colors such that no two adjoining vertices have the same color. Show how to do this using a 3-SAT solver by translating this problem into a 3-CNF formula.

Problem 7:

The monsoon problem:



You are given an array of positive integers. The integers represent the height of a terrain. Assume there is a monsoon-type heavy rainfall. How much water can be stored in the terrain? On the right, the dark-gray boxes represent the heights given by the array [2, 4, 1, 2, 2, 3, 3, 1, 1, 4, 2, 1, 5, 2]. The light-blue boxes represent the stored water, namely, 20. Below is another example with one array value incremented.

Give a algorithm linear in the size of the array that calculates the water that will be stored. Hint: How much water can be stored over an array coordinate?

Problem 8:

A 0-1 knapsack problem with values 4, 5, 6, 9,13,16,19,20,21,25 and weights 2, 3, 3, 4, 5, 6, 7, 8, 8, 9 yields the following table. Find the optimal solution for a total weight of 40, and then for 19, and then for 26.

```
 0 :   0  0  0  0  0  0  0  0  0  0  0
 1 :   0  0  0  0  0  0  0  0  0  0  0
 2 :   0  4  4  4  4  4  4  4  4  4  4
 3 :   0  4  5  6  6  6  6  6  6  6  6
 4 :   0  4  5  6  9  9  9  9  9  9  9
```

```
 5 :   0  4  9 10 10 13 13 13 13 13 13
 6 :   0  4  9 11 13 13 16 16 16 16 16
 7 :   0  4  9 11 15 17 17 19 19 19 19
 8 :   0  4  9 15 15 19 20 20 20 21 21
 9 :   0  4  9 15 19 22 22 23 23 23 25
10 :   0  4  9 15 20 23 25 25 25 25 25
11 :   0  4  9 15 20 26 29 29 29 29 29
12 :   0  4  9 15 24 28 29 32 32 32 32
13 :   0  4  9 15 24 28 33 35 35 35 35
14 :   0  4  9 15 24 32 35 36 36 37 38
15 :   0  4  9 15 24 33 38 39 39 40 41
16 :   0  4  9 15 24 33 39 41 41 41 44
17 :   0  4  9 15 24 37 42 44 44 44 46
18 :   0  4  9 15 24 37 44 48 48 48 48
19 :   0  4  9 15 24 37 44 48 49 50 50
20 :   0  4  9 15 24 37 48 52 52 53 54
21 :   0  4  9 15 24 37 49 54 55 56 57
22 :   0  4  9 15 24 37 49 57 57 57 60
23 :   0  4  9 15 24 37 53 58 59 60 62
24 :   0  4  9 15 24 37 53 61 61 62 65
25 :   0  4  9 15 24 37 53 63 64 65 66
26 :   0  4  9 15 24 37 53 63 68 69 69
27 :   0  4  9 15 24 37 53 67 68 70 73
28 :   0  4  9 15 24 37 53 68 72 73 75
29 :   0  4  9 15 24 37 53 68 74 76 78
30 :   0  4  9 15 24 37 53 72 77 78 81
31 :   0  4  9 15 24 37 53 72 78 80 82
32 :   0  4  9 15 24 37 53 72 81 82 85
33 :   0  4  9 15 24 37 53 72 83 85 87
34 :   0  4  9 15 24 37 53 72 83 89 90
35 :   0  4  9 15 24 37 53 72 87 89 94
36 :   0  4  9 15 24 37 53 72 88 93 95
37 :   0  4  9 15 24 37 53 72 88 95 98
38 :   0  4  9 15 24 37 53 72 92 98 101
39 :   0  4  9 15 24 37 53 72 92 99 103
40 :   0  4  9 15 24 37 53 72 92 102 105
```

Problem 9:

A Pythagorean triple is a triple of natural numbers $a, b,$ and c, such that $a^2 + b^2 = c^2$. You are given an array with n natural numbers. What is the most efficient way to find a Pythagorean triple among the array elements?

(There are a number of solutions, depending on the amount of preprocessing you are willing to do.)

Problem 10:

In how many ways can you write a number n as a sum of ones, twos, and fives? Order counts, i.e. $4 = 1 + 1 + 2 = 2 + 2 = 1 + 2 + 1 = 2 + 1 + 1$ are different ways to write 4.

Problem 11:

You are given a file containing the friend relationship on Facebook. (If I am your friend, then you are also my friend.) How would you test the six degrees of separation conjecture (all people are six or fewer social connections away from each other.) on Facebook.

Problem 12:

You are given a large set of students at a university in Uruguay. Students will have to take examinations in several subjects, after having qualified by passing the corresponding course. You have to make sure that students have no more than one subject exam per day. The university administration asks you to find a schedule with minimum number of exams. You are given a list of students and the subjects in which they want to take the examination. Explain to the administration that this is an NP-complete problem and what that means. How would you go about solving it anyway?

Problem 13:

A Gray code is an enumeration $\left(a_i\right)_{i\in 0,\ldots,2^n-1}$ of the binary numbers from $0 = 000000\ldots 0_b$ to $2^n - 1 = 111\ldots 11_b$ such that a_i and a_{i+1} differ in exactly one binary digit. Also, a_0 and a_{2^n-1} also differ in exactly one binary digit. For example 0, 4, 6, 2, 3, 7, 5, 1 is a Gray code with $n = 3$. How would you find a Gray code given the first five elements of it?

Problem 14:

You are given a maze, consisting of empty and filled in squares. How would you find a good route from the start A to the end B of a maze like the following one. You can move up, down, left, right.