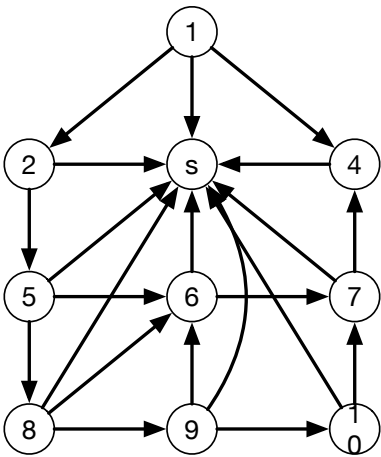


# Extra Credit Homework

Up to 100 points, that can be translated into either  
 20 points for missing midterm and final score points  
 100 points for homework and programming assignment points

**No collaboration, No use of the web!**  
**Violators will have to face final judgement (or me if I find out).**

An *emil* in a directed graph is a node  $s$  such that  $\forall v \in V - \{s\} : (v, s) \in E$  and  $\nexists v \in V : (s, v) \in E$ . This means that all other nodes have an edge towards  $s$  but no nodes has an edge towards  $s$ . An example for a digraph with universal sink is given below:



Assume that a matrix is given by an **adjacency matrix** with a coefficient  $m_{i,j} = 0$  if there is no edge from Node  $i$  to Node  $j$ , and  $m_{i,j} = 1$  if there is an edge from Node  $i$  to Node  $j$ . The adjacency matrix of the above graph is given as

$$(m_{i,j})_{i,j \in \{1,2,\dots,10\}} = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

Give a  $\Theta(|V|)$  time algorithm to determine whether a graph, given by an adjacency matrix, has an emil. Explain why your algorithm works.