Homework 3

Due September 17, 2024 via D2L

Problem 1:

(20 points)

We want to investigate the diophantine equation $x^3 + y^3 + 1 = z^3$. There is an infinite family of solutions, due to Mahler, namely $\left(9k^3 - 1, 9k^4 - 3k, 9k^4\right)_{k \in \mathbb{N}}$, but there are other solutions.

We write a program that searches for all possible solutions $1 < x < y < z$ for a certain range.

What is the number of times the if-statement in the interior block of the following function is executed? Instead of an exact answer, give the asymptotic order.

```
def simple(my_range):
    for x in range(1, my_range):
        for y in range(x+1,my_range):
            for z in range(y+1, 3*my_range):
                if x**3+y**3+1==z**3:
                    print(x, y, z)
```

We can speed up this code by using a function that checks whether $x^3 + y^3 + 1$ is a cube. What is the number of times the function invocation in the interior block of the following function is executed?

```
def get_cube(x):
    guess = int(x**(1/3))
    if guess**3 == x:
        return guess
    elif (guess+1)**3 == x:
        return guess+1
    return None

def better(my_range):
    for x in range(1, my_range):
        for y in range(x+1, my_range):
            a = get_cube(x**3+y**3+1)
            if a:
                print(x, y, a)
```

Problem 2:

(50 points)

Find the asymptotic relationship between the following pairs of functions f, g, i.e. $f \in o(g)$; $f \in \Theta(g)$; $f \in \Omega(g)$ with a calculation showing your reasoning.

(a) $f(n) = n^2 log(n) \qquad g(n) = n^3$

(b) $f(n) = n^3 \quad g(n) = 3^n$

(c) $f(n) = \log(\log(n)) \quad g(n) = (\log(n))^2$

(d) $f(n) = n^2 + n + 1 \quad g(n) = n^2$

(e) $f(n) = n \cdot 3^n \quad g(n) = 3^n$

Problem 3:

(30 points)

Bubblesort can be implemented in Python as

```
def bubble_sort(arr):
    for n in range(len(arr) - 1, 0, -1):
        for i in range(n):
            if arr[i] > arr[i + 1]:
                    arr[i], arr[i + 1] = arr[i + 1], arr[i]
```

Find a loop invariants for the inner and for the outer loop that help establishing its correctness.