Homework 3 Solutions

Problem 1:

The if-statement in the first code is executed $O(N^3)$ times where N is the `my_range` variable. The z-loop executes it $O(n)$ times, and the y-loop executes $O(N^2)$ times.

In the second code, the number of calls to the function is $O(n^2)$, and the function call executes in time $O(1)$.

Problem 2:

(a) $\lim_{n\to\infty} \frac{n^2 \log(n)}{n^3} = \lim_{n\to\infty} \frac{\log(n)}{n} =^{LH} \lim_{n\to\infty} \frac{1/n}{1} = 0$ so $n^2 \log(n) \in o(n^3)$ for $n \to \infty$

(b) $\lim_{n\to\infty} \frac{n^3}{3^n} = \lim_{n\to\infty} \frac{3 \cdot n^2}{\log(3)3^n} =^{LH} \lim_{n\to\infty} \frac{6n}{\log(3)^2 \cdot 3^n} =^{LH} \lim_{n\to\infty} \frac{6}{\log(3)^3 3^n} = 0$, so $n^3 = o(3^n)$ for $n \to \infty$.

(c) $\frac{d \log(\log(n)}{dn} = \frac{1}{n \cdot \log(n)}$, $\frac{d(\log(n))^2}{dn} = \frac{2\log(n)}{n}$, so $\lim_{n\to\infty} \frac{\log(\log(n))}{(\log(n))^2} = \lim_{n\to\infty} \frac{1}{2 \cdot \log(n)^2} = 0$, so $\log(\log(n)) = o(\log(n)^2)$ for $n \to \infty$

(d) $\lim_{n\to\infty} \frac{n^2 + n + 1}{n^2} = \lim_{n\to\infty} \frac{n^2}{n^2} + \frac{n}{n^2} + \frac{1}{n^2} = 1 + 0 + 0 = 1$, so $n^2 + n + 1 \in \Theta(n^2)$ for $n \to \infty$

(e) $\lim_{n\to\infty} \frac{n \cdot 3^n}{3^n} = \lim_{n\to\infty} n = \infty$ so $n \cdot 3^n \in \Omega(3^n)$ for $n \to \infty$.

Problem 3:

```
def bubble_sort(arr):
    for n in range(len(arr) - 1, 0, -1):
        for i in range(n):
            if arr[i] > arr[i + 1]:
                    arr[i], arr[i + 1] = arr[i + 1], arr[i]
```

After an execution of the inner loop, the maximum element of the previous `arr[0], arr[1], … arr[n]`, which is the slice `arr[0:n+1],` is in the last position `arr[n]`.

Before an execution of the outer loop, the elements `arr[n+1], … arr[-1]` are greater or equal than all of the other array elements, and are in order.
After an execution of the outer loop, the elements `arr[n], … arr[-1]` are greater or equal than all of the other array elements, and are in order.