Homework 5

due October 1, 2025

Problem 1:

40 pts

Use the Master Theorem if possible to determine the growth of the functions defined by the following recurrences.

1. $T(n) = 4T(n/3) + \log(n)n.$
2. $T(n) = 3T(n/3) + \sqrt{n}.$
3. $T(n) = 2T(n/2) + n\log(n)$
4. $T(n) = 16T(n/4) + n$

Problem 2:

20 pts

Use induction to show that the recurrence $a_i = 2a_{i-1} + 1$ is solved by $a_i = 2^i - 1.$

Problem 3:

40 pts

(a) Show that it is impossible to find the maximum and minimum of three numbers in less than three comparisons, but that three comparisons are sufficient.

(b) Assume that we try to find minimum and maximum of n numbers. We divide the array into $\lfloor n/3 \rfloor$ groups of three and possible an additional group with one or two numbers. We then calculate the maxima and minima of the groups. If the last group has two numbers, we also determine its maxima and minima. We then obtain the maximum by using comparisons among the maxima and similarly for the minima. Give the number of comparisons that are needed to calculate maximum and minimum of the array elements, based on whether n is divisible by 3, has remainder 1 when divided by 3, or has remainder 2 when divided by 3.