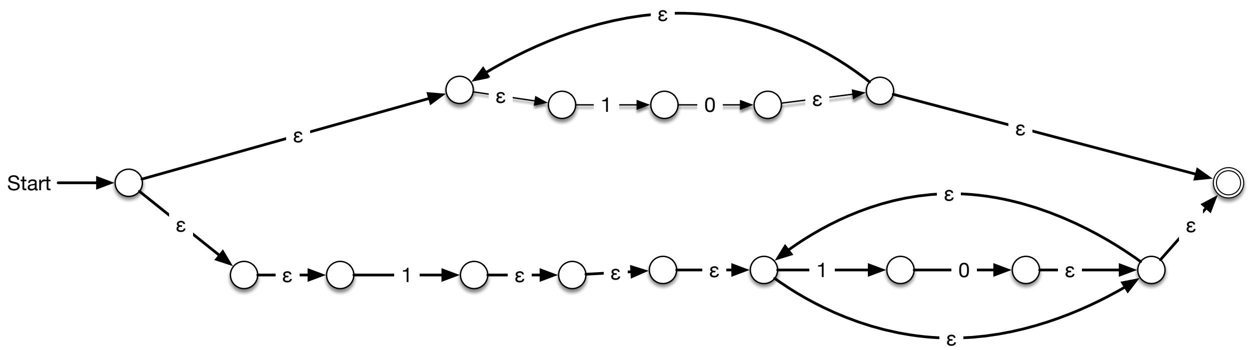
Sample Midterm Solution

Problem 1: You probably have left out many unnecessary ϵ moves.



Problem 2: Let r_n be the number of recursive calls to the function if it is called on input n. Obviously, there are no recursive calls if $n \leq 3$, so that $r_0 = r_1 = r_2 = r_3 = 0$. Otherwise, there are three recursive calls, that in turn will trigger recursive calls. We read off:
$r_n = 3 + r_{n-1} + r_{n-2} + r_{n-3}$.

Problem 3: The loop invariant is vacuously true before the for-loop starts as the sub-array does not have any elements. For the induction step, assume that the loop invariant holds for the iteration with j. Thus, answer $= \max\{A[0], A[1], \ldots, A[j-1]\}$. Assume first that the if-statement does not change answer. This means $A[j] < \max\{A[0], A[1], \ldots, A[j-1]\}$ and therefore $\max\{A[0], A[1], \ldots, A[j-1]\} = \max\{A[0], A[1], \ldots, A[j-1], A[j]\}$. Since answer has not changed, the loop invariant remains true.
Now assume that the if statement changes answer. Then
$A[j] >$ answer $_{\text{before}} = \max\{A[0], A[1], \ldots, A[j-1]\}$
and answer$_{\text{after}} = A[j] = \max\{A[j], \max\{A[0], \ldots A[n-1]\}\} = A[j]$.

Problem 4: Because

$$\lim_{n\to\infty} \frac{\frac{n}{\log(n^2}}{\sqrt{n}} = \lim_{n\to\infty} \frac{\sqrt{n}}{2\log n} = \lim_{n\to\infty} \frac{\frac{1}{2}n^{-1/2}}{n^{-1}} = \lim_{n\to\infty} \frac{1}{2} n^{1/2} = \infty$$

(with an application of L'Hôpital's Rule in the second equality), we have $\dfrac{n}{\log(n^2)} = \Omega\sqrt{n})$.

Problem 5: We need to compare with $n^{\log_3(4)}$. Since $\log_3(4) > 1$,

$$\sqrt{n} = n^{1/2} = O(n^{\log_3(4)-0.01})$$

and we can apply Case 1 of the MT. This means $T(n) = \Theta(\log_3(4))$.