Homework 2 Solutions

Problem 1:

(a) $$\lim_{n\to\infty} \frac{\log(n)^2}{\sqrt{n}} = \lim_{n\to\infty} \frac{2\log(n)\frac{1}{n}}{\frac{1}{2}n^{-\frac{1}{2}}}$$
$$= \lim_{n\to\infty} 4\frac{\log(n)}{n^{\frac{1}{2}}}$$
$$= \lim_{n\to\infty} 4\frac{\frac{1}{n}}{\frac{1}{2}n^{-\frac{1}{2}}}$$
$$= 8\lim_{n\to\infty} n^{-\frac{1}{2}}$$
$$= 0.$$

Therefore, $\log(n)^2 \in o(\sqrt{n})$

(b) $\lim_{n\to\infty} \frac{e^n}{2^n} = \lim_{n\to\infty} \left(\frac{e}{2}\right)^n = \infty$, therefore $e^n \in \Omega(2^n)$.

(c) $\lim_{n\to\infty} \frac{\frac{n^2+1}{n+5}}{3n} = \lim_{n\to\infty} \frac{\frac{1+1/n^2}{1+5/n}}{3} = \frac{1}{3}$, therefore $\frac{n^2+1}{n+5} \in \Theta(3n)$.

Problem 2:

This amounts to selecting 8 out of 64 squares, or $\binom{64}{8}$ or 4426165368 possibilities.

Problem 3:

Each three-by-three grid has $9!$ possibilities of putting in nine numbers. The total number is

109110688415571316480344899355894085582848000000000

or

1.0911068841557131e+50.

Thus, there are 51 decimal digits.

Problem 4:

We just adorn the Euclidean Algorithm code with a print statement and obtain:

```
def gcd(a,b):
    if b == 0:
        return a
    print(f'gcd({a},{b})')
```

```
    return gcd(b, a%b)
```

This gives us:

```
gcd(779625000,330115500)
gcd(330115500,119394000)
gcd(119394000,91327500)
gcd(91327500,28066500)
gcd(28066500,7128000)
gcd(7128000,6682500)
gcd(6682500,445500)
gcd(445500,0)
```