Homework 3

Problem 1:

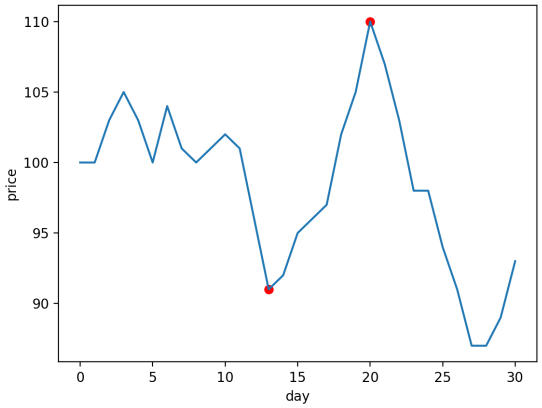
Use mathematical induction to show that the recurrence $a_{i+1} = 2a_i + 1$ with $a_0 = 0$ is solved by $a_i = 2^i - 1$.

Problem 2:

Use mathematical induction to show that the solution of $a_{n+1} = 3 \cdot a_n + n$ with $a_0 = 0$ is solved by $a_n = \frac{1}{4}\left(-1 + 3^n - 2n\right)$.

Problem 3:

You are given the price of a commodity at the end of the trading day in an array with some 20 values corresponding to the trading days in a given month. You are using this data to test an AI program that either does not make any trade during a month or makes one purchase and later one sale. For your program, you need to determine the best day to purchase the commodity and the best day to sell the commodity during the month. You are not allowed to sell what you do not have and you are only allowed one buy and one sale, though you might decide not to buy, e.g. because the commodity price is always falling. Of course, you want to find the days that maximize the profit. On the right, there is an example.



(a) Show by example that the difference between the maximum of the array and the minimum is not always the maximum profit that can be realized.
(b) The brute-force method tests all buy days and all sales days (necessarily after the buy date). What is the number of potential profits that the brute-force method will try out? Implement it.
(c) A better way is to use a divide-and-conquer approach, where you divide the trading interval into a left and a right half. It is possible that the buy and sell day are in the same half or that the buy day is in the left half and the sell day in the right half. Give the recurrence formula for the runtime.

Please submit also a Python file with your code for (b) and (c).