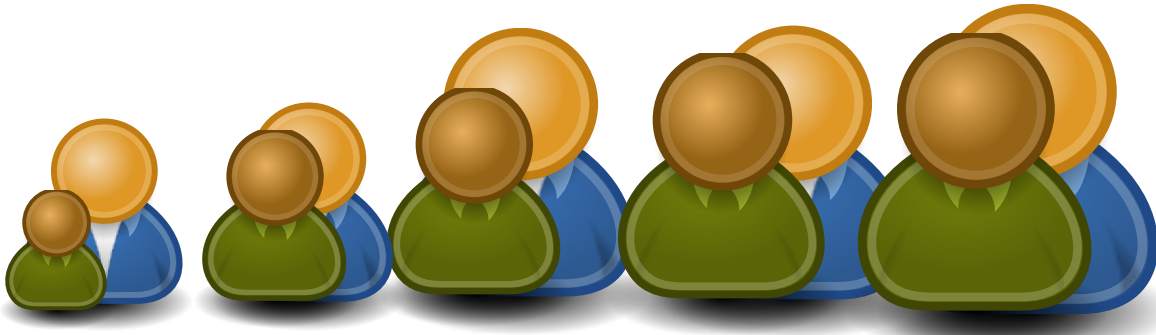# Homework 5

Problem 1:

You are taking pictures of teams at an indoors soccer tournament. The organizer wants the teams to stand in two rows, with one team in front of the other. The organizer also insists that the player standing behind the player is taller than the player in front of them. Before the game, you are given the heights of each team (in cm) as an array $[a_1, a_2, \ldots, a_n]$ and $[b_1, b_2, \ldots, b_n]$.

You have a valid placement of both teams if after reordering $a_1 < b_1, a_2 < b_2, \ldots, a_n < b_n$.
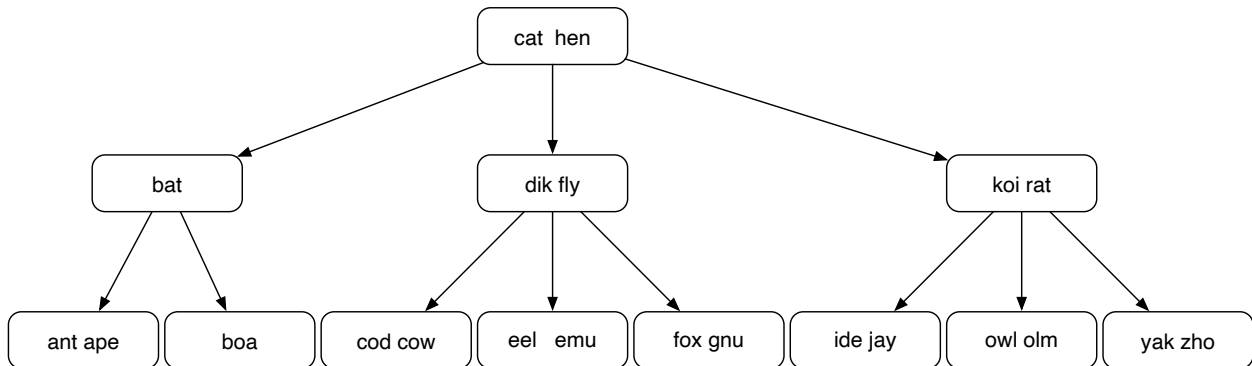
(a) How many possible ways are there to order the two teams (with n players each)?

(b) Show that if the photo is possible, then there is one way where the players in the back row are ordered by height, from left to right.

(c) Show that is the photo is possible, then we can have a photo where the players in the front are also ordered.

Problem 2:

(a) Show that it is impossible to find the maximum and minimum of three numbers in less than three comparisons, but that three comparisons are sufficient.

(b) Assume that we try to find minimum and maximum of n numbers. We divide the array into $\lfloor n/3 \rfloor$ groups of three and possible an additional group with one or two numbers. We then calculate the maxima and minima of the groups. If the last group has two numbers, we also determine its maxima and minima. We then obtain the maximum by using comparisons among the maxima and similarly for the minima. Give the number of comparisons that are needed to calculate maximum and minimum of the array elements, based on whether n is divisible by 3, has remainder 1 when divided by 3, or has remainder 2 when divided by 3.
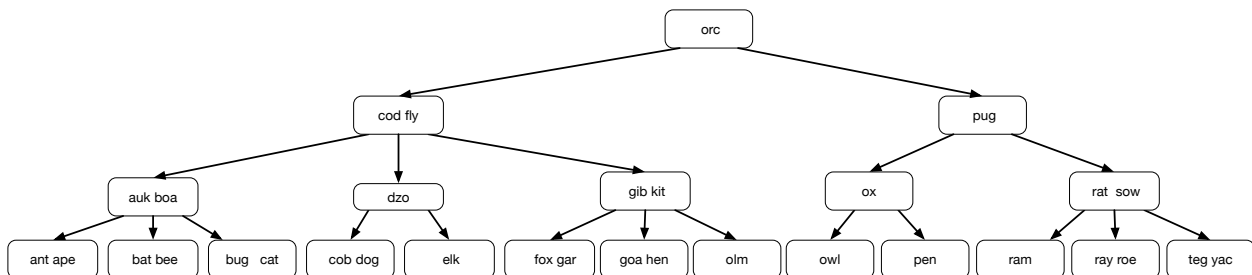
Problem 3:

Insert into the following B-tree. (Use rotations if possible and prefer left rotation over right rotation.) Show all steps.

```
                          ┌─────────┐
                          │ cat  hen │
                          └─────────┘
              ┌──────────────┼───────────────────┐
         ┌─────┐         ┌───────┐           ┌────────┐
         │ bat │         │ dik fly │          │ koi rat │
         └─────┘         └───────┘           └────────┘
        ┌────┴────┐     ┌────┬────┐       ┌──────┼──────┐
   ┌────────┐ ┌─────┐ ┌───────┐ ┌───────┐ ┌────────┐ ┌────────┐ ┌────────┐
   │ ant ape│ │ boa │ │cod cow│ │eel emu│ │fox gnu │ │ ide jay│ │ owl olm│ │ yak zho│
   └────────┘ └─────┘ └───────┘ └───────┘ └────────┘ └────────┘ └────────┘ └────────┘
```

(a) Insert 'gib' into the original tree.

Problem 4:

From the following B-tree, use delete operations. (For deletes, always choose the predecessor and prefer left rotate over right rotate).

```
                                    ┌─────┐
                                    │ orc │
                                    └─────┘
                   ┌──────────────────┴────────────────────┐
              ┌────────┐                                 ┌─────┐
              │ cod fly │                                │ pug │
              └────────┘                                 └─────┘
         ┌────────┼──────────┐                      ┌──────┴──────┐
   ┌────────┐ ┌─────┐  ┌────────┐              ┌──────┐      ┌────────┐
   │ auk boa│ │ dzo │  │ gib kit│              │  ox  │      │ rat sow│
   └────────┘ └─────┘  └────────┘              └──────┘      └────────┘
   ┌───┬───┬───┐  ┌───┬───┐  ┌───┬───┬───┐   ┌───┬───┐   ┌───┬───┬───┐
```

| ant ape | bat bee | bug cat | cob dog | elk | fox gar | goa hen | olm | owl | pen | ram | ray roe | teg yac |

1. First delete 'orc'.
2. Then delete 'roe' from the resulting 2-3 tree.
3. Then delete 'rat' from the resulting 2-3 tree.

Explain all steps and show the tree after each deletion.