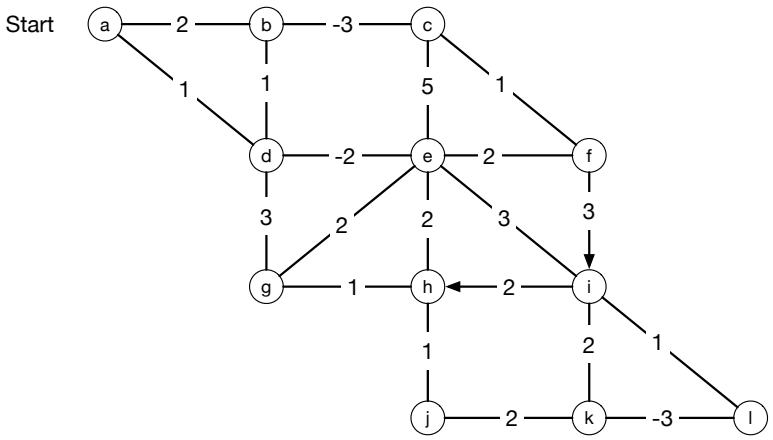
Homework 9

due April 13, 2025

Problem 1:

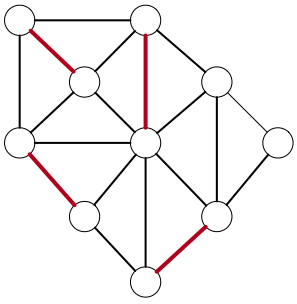
The following graph has a negative cycle. What happens if we apply Dijkstra's algorithm to it? We are trying to determine distances from Node a.

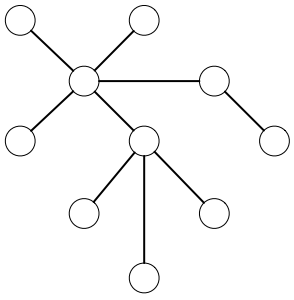


Problem 2:

A perfect matching on an undirected graph with $2n$ vertices is a set of n edges such that each vertex belongs to exactly one of these edges. The thicker, reddish edges in the following graph show a perfect matching. Normally, it is difficult to find perfect matchings or determine that no perfect matching exists. This is not the case for trees. A tree is a connected graph of n vertices and $n - 1$ edges, that does not have a cycle. Show that the following algorithm finds a perfect matching:

> Select a node of degree 1 (i.e. an endpoint). Match the node to its unique neighbor. Then delete both nodes. Continue. If at any point you cannot find a node of degree 1, output "No perfect matching". If no more points are left, you have found a perfect matching.





Prove that this algorithm is correct. Give an example of a tree that does not have a perfect matching.