

$$\leq \sqrt{n} \cdot C \sqrt{n \log(\log(\sqrt{n}))} + n$$