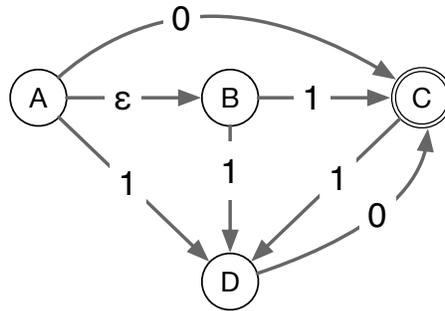


Midterm

Submit solutions to exercises for a total of 80 points maximum.

- 10 points (1) What are the transitions in the DFA from State $\{A, B\}$ during the process of constructing a DFA from the NFA depicted below.



- 10 points (2) Which is the smallest string that satisfies the regular expression $(01)^+0^*(0+1)^*1$.

- 10 points (3) What is the run-time of the following algorithm (as a Theta of a power of n):

```
def temp(n):
    result = 0
    for i in range(n):
        if i%2:
            result *= 2
        else:
            for j in range(n**2):
                result += j**2
    return result
```

- (4) In order to simultaneously determine the minimum and maximum of an array, the following procedure is used. First, we divide the array into m groups of four (and possibly one overflow group with less than four elements). We then use four comparisons per full group in order to determine the maximum and the minimum of the group. Finally, we make $m - 1$ comparisons in order to find the maximum of the maxima and $m - 1$ comparisons in order to find the minimum of the minima. What is the number of comparisons if

10 points

(a) the number n of elements in the array is a multiple of four.

10 points

(b) the number n of elements in the array satisfies $n \equiv 2 \pmod{4}$.

Compare with the number of comparisons given in lecture of $3n/2 - 2$.

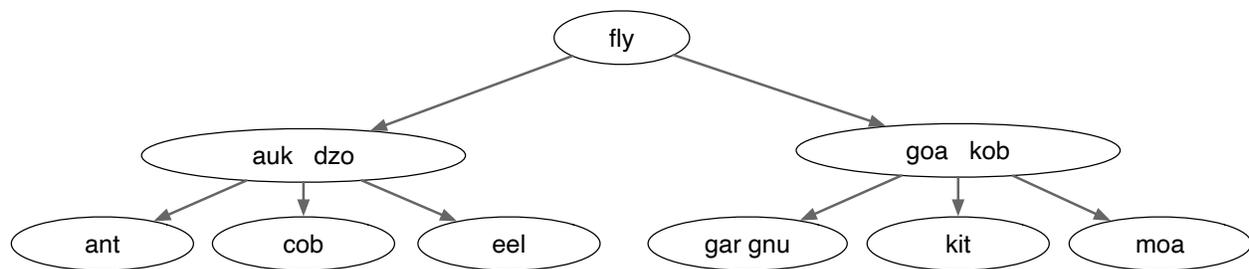
- (5) A sorting algorithm subdivides an array into sub-arrays of one fifth of the length of the original array. The sorting algorithm then:
- (1) Concatenates the first two sub-arrays, sorts them recursively, and splits them.
 - (2) Concatenates the second and third sub-array, sorts them recursively, and splits them.
 - (3) Concatenates the third and fourth sub-array, sorts them recursively, and splits them.
 - (4) Concatenates the fourth and fifth sub-array, sorts them recursively, and splits them.
 - (5) Concatenates the first two sub-arrays, sorts them recursively, and splits them.
 - (6) Concatenates the second and third sub-array, sorts them recursively, and splits them.
 - (7) Concatenates the third and fourth sub-array, sorts them recursively, and splits them.
 - (8) Concatenates the fourth and fifth sub-array, sorts them recursively, and splits them.
- Of course, for very small arrays, it uses a different sorting algorithm such as bubble-sort. Give a recursion formula for the run-time of this algorithm and solve it using the Master Theorem in the book.

10 points
10 points

10 points

10 points

- (6) What is the split number and the level in a Linear Hashing structure with 20 buckets? In which bucket would a record with hash of key 40 go?
- (7) Describe the result of deleting cob from the following small 2-3 B-tree?



10 points

- (8) You are given a jar full of red or blue marbles. You also have an unlimited amount of red marbles on the side. You then repeatedly execute the following procedure:

```

def procedure():
    Randomly select two marbles from the jar, called m1 and m2
    If m1 and m2 have the same color:
        Remove the marbles from the jar and place a red marble in the jar instead
    If m1 and m2 have different colors:
        Place the blue marble back in the jar and toss the red marble
    Continue until only one marble is left in the jar.
  
```

10 points

10 points

10 points

- (1) Explain why this procedure will end up with only one marble in the jar.
- (2) Show that the parity (even/odd) of the number of blue marbles in the jar does not change in one step of the procedure.
- (3) Show that if the number of blue marbles is even, the last marble is red.