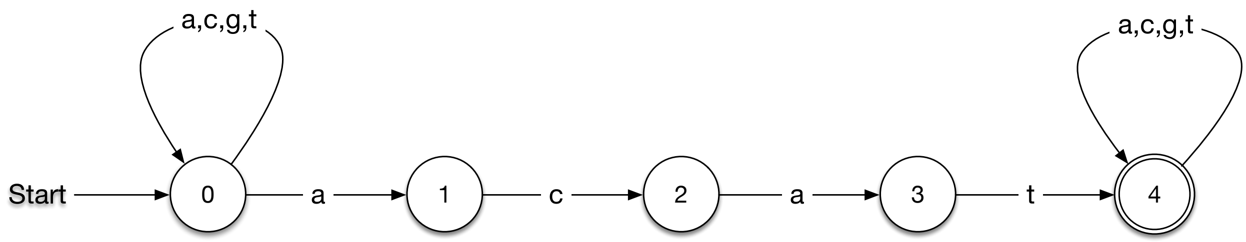
Homework 3 Algorithms

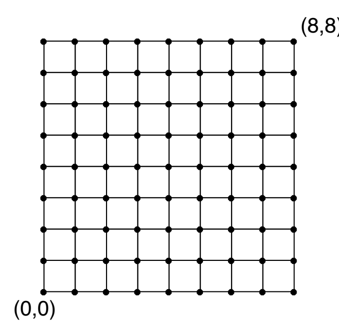
1. Converting an NFA to a DFA:
Use the construction in the slides to convert the following NFA to a DFA. The NFA recognized all strings containing "acat" in the alphabet Σ={a,c,g,t} of DNA strings. It basically decides sometimes on seeing an "a" in the initial state that this might be the beginning of the pattern. This is actually the only feature of non-determinism.



2. Use mathematical induction to show that the number of subsets of a set with n elements is 2^n.

3. We want to count the number of paths along grid-lines of an $n \times n$ grid. so that the path never goes below the diagonal.
    1. A path starts in (0,0) and ends in (n,n)
    2. A path-segment is along the edges of a constituting square and can either go up or go left.
    3. A path is continuous.
    4. A path never touches a grid point (i, j) with $i > j$.

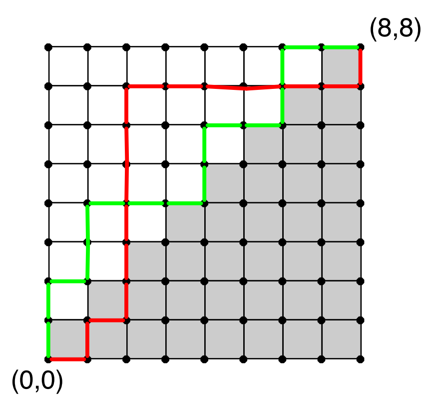
The grid



A good path (green) and a bad path (red):

The red path touches (1,0), which is forbidden.

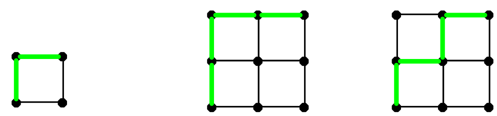
The green path touches (0,0), (0,1), (0,2), (1,2),



(1,3), (1,4), (2,4), (3,4), (4,4), (4,5), (4,6), (5,6),
(6,6), (6,7), (6,8), (7,8), (8,8).

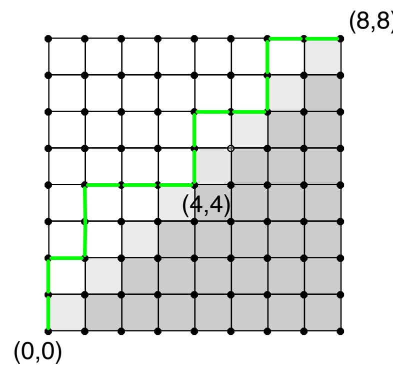
A good path never crosses the main diagonal, but a bad path does so. Let p_n denote the number of good paths in an $n \times n$ grid.

p_0 is equal to 1, since there is only one path, which happens to be good. $p_1 = 1$, since there are two paths, and only one is good. $p_2 = 2$, as the following drawing shows:



(A) Let q_n be the number of paths that are good, but never even touch the diagonal. Use a picture to show that $q_n = p_{n-1}$.

(B) Let *k* be the first time that a good path touches the diagonal. (Since it ends on a diagonal point, there has to be such a *k*.) Show that the number of such paths is $q_k p_{n-k}$.



(C) Use this result to give a recurrence relation for p_n in terms of $q_1, p_{n-1}, q_2, p_{n-2}, \ldots, q_n, p_0$.