Laboratory 1

Problem 1:
The solutions of a quadratic equation $ax^2 + bx + c = 0$ are given by

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

so the solution can be a complex number is the number under the square root (known as the discriminant) is negative.

Write a program that asks the user to enter the values for a, for b, and then for c. The program then prints the two solutions. Use exponentiation by 0.5 to calculate the square root. If the "discriminant" $b^2 - 4ac$ is negative, Python will happily calculate and return complex solutions.

Problem 2:
Write a program that takes a ship's speed in nautical miles per day and converts it into miles per hour.

Problem 3:
The Babylonian method to calculate a square root has become known as Heron's method named after a Greek mathematician from Alexandria in the Nile delta. If S is the number for which we want to calculate $\sqrt{S}$, we proceed with an initial guess x. We then improve the guess iteratively by updating the guess by the mean of x and S/x, i.e. by setting

$$x = \frac{1}{2}(x + \frac{S}{x}).$$

Write a program that asks the user to enter S. Then starting with guess $x = S/2$, apply the update five times. Then print out the result. You can also compare your result with the true value.

What you will find:
For small numbers, Heron's method works really well. For larger numbers, the initial guess is too far off and it takes many more steps then we to get close to a useful approximation of the root. If the number is negative, Heron's method will yield guesses that diverge. This underlies the need for a programming language to react to the result of its calculation.

Problem 4:
Heron's method is just one of many numerical algorithms that proceeds by iteratively improving a guess. The Raphson-Newton method solves an equation $f(x) = 0$ by starting with an initial guess x an improving it by calculating

$$x = x - \frac{f(x)}{f'(x)}.$$

If we apply this method to the equation $x^3 + x + 1 = 0$, we improve our guess by

$$x = x - \frac{x^3 + x + 1}{3x^2 + 1}.$$

Your task is to write a program that asks the user for an initial guess and then updates the value of the guess seven times before printing it out.