# Algebraic and Logical Query Languages 

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## Bags, Lists, Sets

- Bags are multi-sets
- An element can appear more than once
- They are not sets
- In a set, each element can appear at most once
- They are not lists
- In a list, elements are indexed


## Bags, Lists, Sets

- Why bags:
- Union, seletion and projection can create the same tuple many times
- Removing duplicates is difficult:
- Either use a hash table or use sorting
- Both of which are expensive in different ways


## Bags, Lists, Sets

- Why bags:
- For some temporary tables, bags are appropriate
- Aggregation query like find the average salaries of all female employees hired in 2010, 2011, 2012
- Form a temporary table with salary as only attribute
- You need to keep values separate


## Union, Intersection, Differences of Bags

- Union:
- Just concatenate the two bags
- If an element appears twice in one bag and thrice in the other, it will appear five times in the union


## Union, Intersection, Differences of Bags

- Intersection
- $R \cap S$ :
- Bags match each tuple with another tuple
- If a tuple appears $n$ times in $R$ and $m$ times in $S$, then it appears $\min (m, n)$ times in $R \cap S$.


## Union, Intersection, Differences of Bags

- Difference:
- Again, bags use one-to-one matching
- Tuple appears $n$ times in $R$
- Tuple appears $m$ times in $S$
- Tuple appears $\max (0, n-m)$ times in $R-S$.
- Each occurrence in $S$ cancels out a single appearance in $R$


# Union, Intersection, Differences of Bags 

- In short: bags are different from sets


## Projection of Bags

- Projection of bags:
- Each tuple in the mother relation gives rise to one tuple in the projection


## Selection of Bags

- Again: selection condition is applied to each tuple
- There is no duplicate elimination


## Products of Bags

- Recall: Product assumes that attribute sets are different
- Each tuple of $R$ is paired with each tuple of $S$


## Joins of Bags

- Join tuple by tuple


## Joins of Bags

- Example:

| $R$ | $\mathbf{A} \mathbf{B}$ | $S$ B C |
| ---: | ---: | ---: |
| 12 | 23 |  |
| 12 | 45 |  |
|  |  | 45 |

$$
R \times S \quad R \bowtie S
$$

## Joins of Bags

- Example:

| $R$ | $\mathbf{A} \mathbf{B}$ | $S$ B C |
| ---: | ---: | ---: |
| 12 | 23 |  |
| 12 | 45 |  |
|  |  | 45 |


| A R.B S.B C |  |  |  | $R \bowtie S \begin{array}{r} \text { A B C } \\ 123 \end{array}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 2 | 3 |  |  |  |
| 1 | 2 | 4 | 5 |  |  | 23 |
| 1 | 2 | 4 | 5 |  |  |  |
| 1 | 2 | 2 | 3 |  |  |  |
|  | 2 | 4 | 5 |  |  |  |
|  | 2 | 4 |  |  |  |  |

## Joins of Bags

| $R$ | $\mathbf{A} \mathbf{B}$ | $S$ B C |
| ---: | ---: | ---: |
| 12 | 23 |  |
| 12 | 45 |  |
|  |  | 45 |

$$
\begin{array}{r}
R \bowtie_{R . B<S . B} S \\
\qquad
\end{array}
$$

## Joins of Bags

| $R$ | $\mathbf{A} \mathbf{B}$ | $S$ B C |
| ---: | ---: | ---: |
| 12 | 23 |  |
| 12 | 45 |  |
|  |  | 45 |

$$
5
$$

## Relational Algebra Operators

- Deduplication operator $\delta$
- Aggregation operators such as sum, averages are used by grouping operators
- Grouping: Partitions tuples into groups
- Usually, aggregation is then applied to each group
- Extended projections
- Allow to create new attributes using arithmetic operations
- Sorting operator
- Outer join operator


## Aggregations

- SUM
- AVG
- MIN, MAX
- COUNT
- not necessarily distinct values in a column


## Aggregation

- Example:
- Find the aggregations of this table

| A B |
| :--- |
| 12 |
| 3 |
| 1 |
| 1 |
| 12 |

## Aggregation

- Example:
- Find the aggregations of this table

| $\mathrm{A} \mathbf{B}$ |
| :--- |
| 12 |
| 3 |
| 12 |
| 12 |

$\left.\begin{array}{llll}\operatorname{SUM}(A) & =6 & & \operatorname{SUM}(B) \\ \text { AVG (A) } & =1.5 & \operatorname{AVG}(B) & =2 \\ \operatorname{MIN}(A) & =1 & \operatorname{MIN}(B) & =2 \\ \operatorname{MAX}(A) & =3 & \operatorname{MAX}(B) & =4 \\ \operatorname{COUNT}(A) & =4 & & \operatorname{COUNT}(B)\end{array}\right)=4$

## Grouping

- Find the length of all movies produced by a certain studio
- Project onto studio, length
- Group by studioName

| studioName | movieLength |
| :--- | :---: |
| Disney <br> Disney <br> Disney | 89 <br> 103 <br> 132 <br> Disney |
| MGM | 89 |
| MGM | 103 |
| MGM | 89 |

## Grouping

- Find the length of all movies produced by a certain studio
- Project onto studio, length
- Group by studioName
- Aggregate on movieLength



## Grouping Operator

- $\gamma_{\mathrm{op}(A)}(R)$
- $A$ - the grouping attribute
- op - the aggregation operator (e.g. AVG)
- $R$ - the relation


## Grouping operator

- $\gamma_{\mathrm{op}(A)}(R)$
- Partition the tuples of $R$ into groups according to values of $A$
- For each group produce one tuple with
- the grouping attributes' values for that group
- the aggregation over all tuples of that group
- Generalize to several attributes


## Grouping Operator

- Find all stars that appeared in at least three movies and the earliest year in which they appeared
- $\gamma_{\text {starName,MIN(year) } \rightarrow \text { minYear,COUNT(title) } \rightarrow \text { ctTitle }}$ (StarsIn)
- Result has starName, minYear, and ctTitle attributes
- Then select based on the last attribute: ctTitle $\geq 3$
- Finally project onto starName and minYear


## Extended Projection Operator

- Classic projection $\pi_{L}(R)$
- $L$ - set of attributes of $R$
- Extended projection $\pi_{L}(R)$
- $L$
-     - single attributes (as before)
-     - expressions $x \rightarrow y$ renaming attribute $x$ to $y$
-     - expressions $E \rightarrow z$ where $E$ is an expression in terms of attributes and operators


## Extended Projection Operator

- Example

| A B C |  |
| :--- | :--- |
| 0 | 1 |$\quad \pi_{A, B+C \rightarrow X}(R)$

## Extended Projection Operator

- Example

| A B C |  |
| :--- | :--- |
| 0 | 1 |
|  | 1 |
|  | 1 |
|  | 4 |

$\pi_{A, B+C \rightarrow X}(R)$

| $A$ | $X$ |
| :--- | :--- |
| 0 | 3 |
| 0 | 3 |
| 3 | 9 |

## Sorting Operator

- $\tau_{L}(R)$
- $L$ is a list of attributes
- Result is $R$ but ordered according to the list $L$


## Outer Join Operator

- Inner join leaves out certain tuples
- Outer join includes them with null values added


## Outer Join Operator

- Example

| $R$ | A B C | $S$ | B C D |
| ---: | :--- | ---: | :--- |$\quad R \stackrel{o}{\bowtie} S$

## Outer Join Operator

- Example

$R \stackrel{o}{\bowtie} S$

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 10 |
| 1 | 2 | 3 | 11 |
| 4 | 5 | 6 | NULL |
| 7 | 8 | 9 | NULL |
| NULL | 6 | 7 | 12 |

## Outer Join Operator

- Left outer join:
- Only dangling tuples in the left relation are padded with NULL and added to the relation
- Right outer join:
- Only dangling tuples in the right relation are padded with NUMM and added to the relation


## Outer Join Operator

- Example

| $R$ | A B C | $S$ | B C D | $R \stackrel{o}{\bowtie}_{l} S$ |
| ---: | :--- | ---: | :--- | ---: |
| 123 |  | 2310 |  |  |
| 456 |  | 2311 |  |  |
| 7 | 89 |  | 6712 |  |

## Outer Join Operator

- Example



## Outer Join Operator

- Example

$R \stackrel{o}{\bowtie_{r}} S$

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 10 |
| 1 | 2 | 3 | 11 |
| NULL | 6 | 7 | 12 |

## Outer Join Operator

- Can also be extended to theta joins

