Sample Midterm Solutions

Problem 1:

According to Shannon's formula, with C denoting achievable channel capacity in bits per second, B the bandwidth of the line, S the average signal power, and N the average noise, we have

$$C = B \cdot \log_2(1 + S/N).$$

The signal to noise ratio is given in decibels, so we need to translate it back to absolute numbers. We have

$$10 \cdot \log_{10}(S/N) = 20\text{dB}.$$

Thus, $\log_{10}(S/N) = 2$ and therefore

$$S/N = 10^2 = 100$$

Since the bandwidth is 20 Mhz, the achievable channel capacity is 133 Mbps.

Problem 2:

Attenuation at this frequency over 1000 m is going to be ten times (1000m/100m=10) the attenuation over 100m, which is 93 dB. This alone suggests that this is not do-able.

Using Shannon's formula again,

$$C = B \cdot \log_2(1 + S/N),$$

we find at best

$$5 \text{ Mbps} = 20 \text{ MHz} \cdot \log_2(1 + S/N),$$

which gives

$$\frac{1}{4} = \log_2(1 + S/N),$$

$$2^{\frac{1}{4}} = 1 + S/N,$$

$$1.18921 = 1 + S/N,$$

$$S/N = 0.18921.$$

This is impossible for a signal, so we set arbitrarily S/N = 2. The signal then needs to be at least 10μW. If A is the attenuation in decibel, E_1 the power at the source, and E_2 the power at the destination, then

$$A = 10 \log_{10}(E_1/E_2).$$

Thus

$$9.3 = \log_{10}(E_1/E_2),$$

or

$$E_2 = 10^{9.3} \times 10\mu\text{W} = 19{,}952.6\text{W}.$$

This power would blow out an average household fuse and heat up the device rather quickly.

Problem 3:

We multiply the received message with H. The result is $(1,1,1,0)$. We look up the column $(1,1,1,0)$ in H and find that it is the tenth column (starting counting with one as the left-most column). We flip the corresponding bit in [1,0,0,1,1,1,0,0,0,**1**,1,1,0,0,0] to obtain

[1,0,0,1,1,1,0,0,0,**0**,1,1,0,0,0]

as the presumed code word sent. As the last four bits of each code word are parity bits, the presumed message was

[1,0,0,1,1,1,0,0,0,0,1].

Problem 4:

A hypothetical SONET SPE spanning two frames would have size $2 \times 9 \times 86$ B (though that is debatable and any reasonable assumption would be accepted). Since the SPE can contain 1548B and the new format takes 153.3 B (on average), we should be able to accommodate 10 of these frames. Per second, we have 8000 SONET frames, so we can send 40,000 TS1 frames per second, for a data rate of 40,000 x 150 B/sec = 6MBps = 48 Mbps.

Problem 5:

Using Aloha, there is no minimum frame size. Frames can be destroyed and this is noticed when there is no acknowledgment. (This is a completely sufficient answer.)

Using CSMA with collision detection, the length of a frame needs to be at least twice the maximum propagation delay. In a CAT-7 twisted pair at 500 m, this gives for the maximum propagation delay τ

$$\frac{500 \text{ m}}{\tau} = 0.75 \times 300 \times 10^6 \text{ m/s}$$

or

$$\tau = \frac{500}{0.75 \times 300 \times 10^6} \text{ sec} = 2.2222 \times 10^{-6} \text{ sec.}$$

A frame with b bits takes $b \times 10^{-7}$ sec to be put on the wire. The minimum frame has therefore 45 bits.

Problem 6:

We first assign 1 to Nodes B and G. This adds edges (A,B) and (A,G) to the tree. We then assign 2 to D and H. This adds edges (G,D) and (G,H). We now assign 3 to E and I. This adds (H,I) and (D,E) to the spanning tree. Then we assign 4 to C, F, and J. this adds (E,C), (E,F), and (E,J) to the spanning tree.