Time Series 2

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Time Series

- General Model of a time series:
  - Trend + seasonal(s) + remainder
  - Trend × seasonal(s) × remainders

- Remainder can be modeled as a random walk

Trends

- Example: Disposable Personal Income Massachusetts and Missouri
  - from Federal Reserve Bank of St. Louis
  - Need to check the raw data: Use separator

```
df_ma = pd.read_csv('MAPCPI.csv',
                     sep = ',',
                   )
df_ma.set_index('DATE', inplace = True)
```

Trends

- Example continued:
  - We have two different files that we want to combine
    - Pandas has a merge function
      - Which needs to have a common column
        - Actually, merge implements an SQL-like join

```
df = pd.merge(df_ma, df_mo, on='DATE')
```

Trends

- We can now use linear recursion

```
from statsmodels.formula.api import ols

…

model = ols("df.MOPCPI ~ df.MAPCPI", df).fit()
inter, coef = model.params
print(inter, coef)
print(model.summary())
```

```
900.173966439816 0.6894112036215065
```

Trends

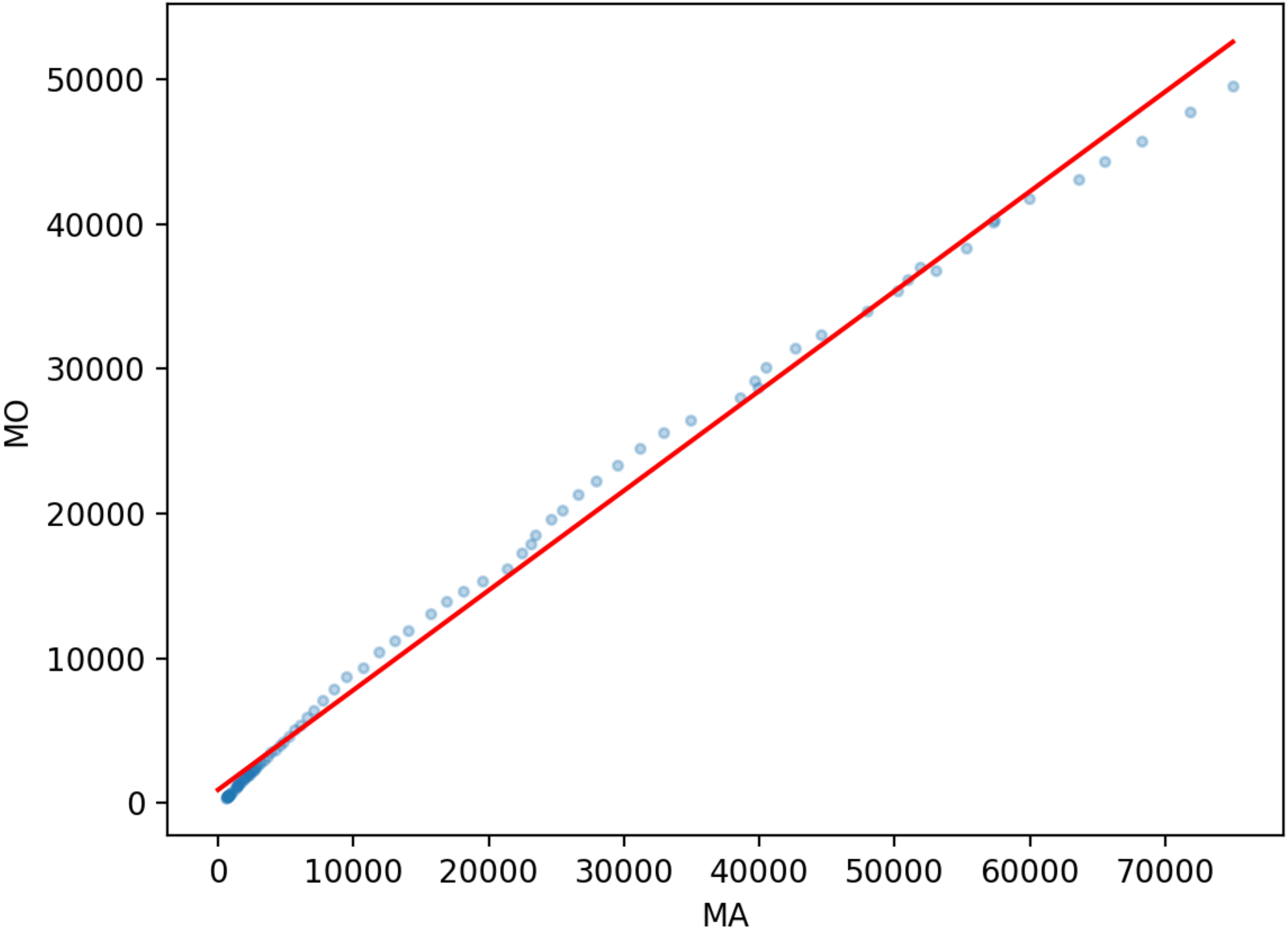
```
                            OLS Regression Results
==============================================================================
Dep. Variable:              df.MOPCPI   R-squared:                       0.995
Model:                            OLS   Adj. R-squared:                  0.995
Method:                 Least Squares   F-statistic:                 1.710e+04
Date:                Fri, 03 Jul 2020   Prob (F-statistic):         1.60e-103
Time:                        16:40:01   Log-Likelihood:                -762.99
No. Observations:                  91   AIC:                             1530.
Df Residuals:                      89   BIC:                             1535.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept    900.1740    147.748      6.093      0.000     606.601    1193.747
df.MAPCPI      0.6894      0.005    130.771      0.000       0.679       0.700
==============================================================================
Omnibus:                        0.064   Durbin-Watson:                   0.066
Prob(Omnibus):                  0.969   Jarque-Bera (JB):                0.069
Skew:                           0.051   Prob(JB):                        0.966
Kurtosis:                       2.911   Cond. No.                     3.69e+04
==============================================================================
```

Trends

- We then plot the result:

```
plt.plot(df.MAPCPI, df.MOPCPI, '.', alpha=0.3)
plt.plot(np.linspace(0,75000), inter +
coef*np.linspace(0,75000), 'r-')

plt.xlabel('MA')
plt.ylabel('MO')
plt.show()
```


50000
40000
30000
20000
10000
0
MO
0
10000
20000
30000
40000
50000
60000
70000
MA

Trends

- We now look at the residual
  - We define it as an additional column

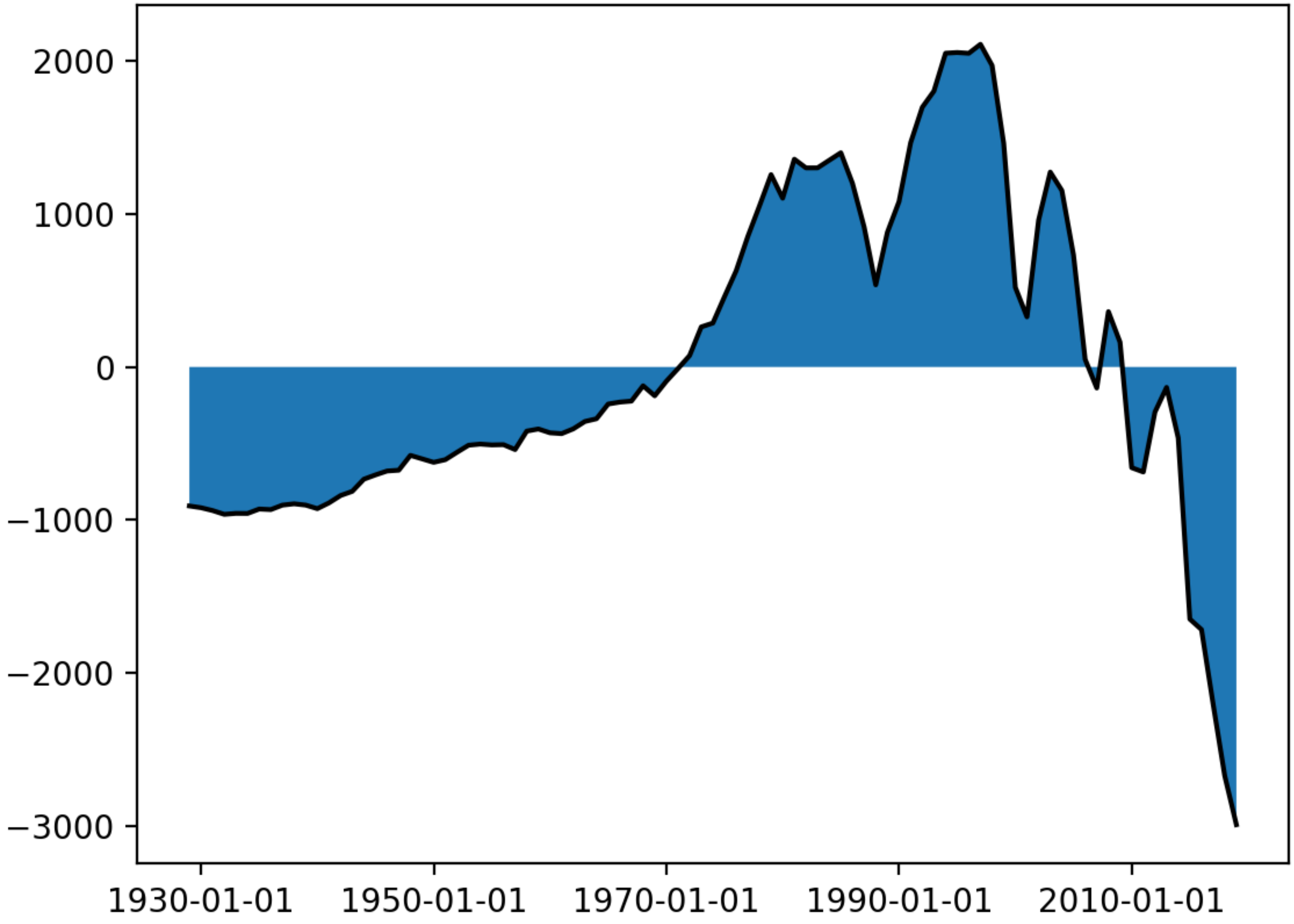
```
df['residual'] = df.MOPCPI - inter-coef*df.MAPCPI
```

Trends

- Showing the residual:
    - Use xticks to make the x-axis readable
    - And use fill to make the result clearer

```
plt.plot(df.residual, 'k-')
plt.xticks(['1930-01-01', '1950-01-01', '1970-01-01',
'1990-01-01', '2010-01-01'])
plt.fill_between(df.index, 0, df.residual)
plt.show()
```

Trends


2000
1000
0
−1000
−2000
−3000
1930-01-01
1950-01-01
1970-01-01
1990-01-01
2010-01-01

Trends

- This would be a bad regression for prediction!

Seasonal Dummy Variables

- Suppose you want to use linear regression
- Need to account for the effect of week-days
  - Introduce dummy variables

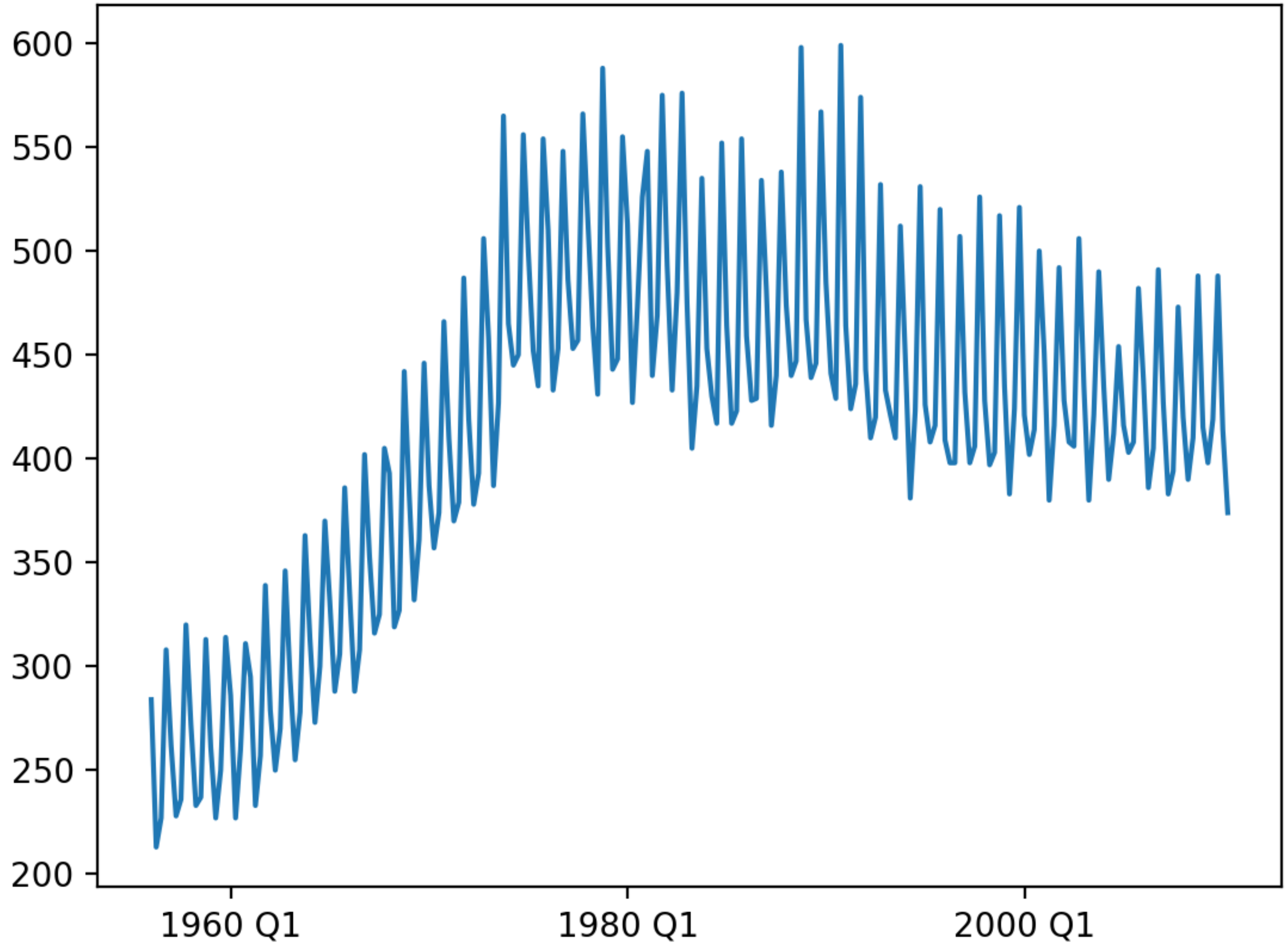
	t1	t2	t3	t4	t5	t6	t7
Mon	1	0	0	0	0	0	0
Tue	0	1	0	0	0	0	0
Wed	0	0	1	0	0	0	0
Thu	0	0	0	1	0	0	0
Fri	0	0	0	0	1	0	0
Sat	0	0	0	0	0	1	0
Sun	0	0	0	0	0	0	1

Seasonal Dummy Variables

- Use linear regression including the dummy variables
  - $y = \beta_0 + \beta_1 x_1 + \ldots \beta_n x_n + \gamma_1 t_1 + \gamma_2 t_2 + \gamma_3 t_3 + \ldots + \gamma_7 t_7$

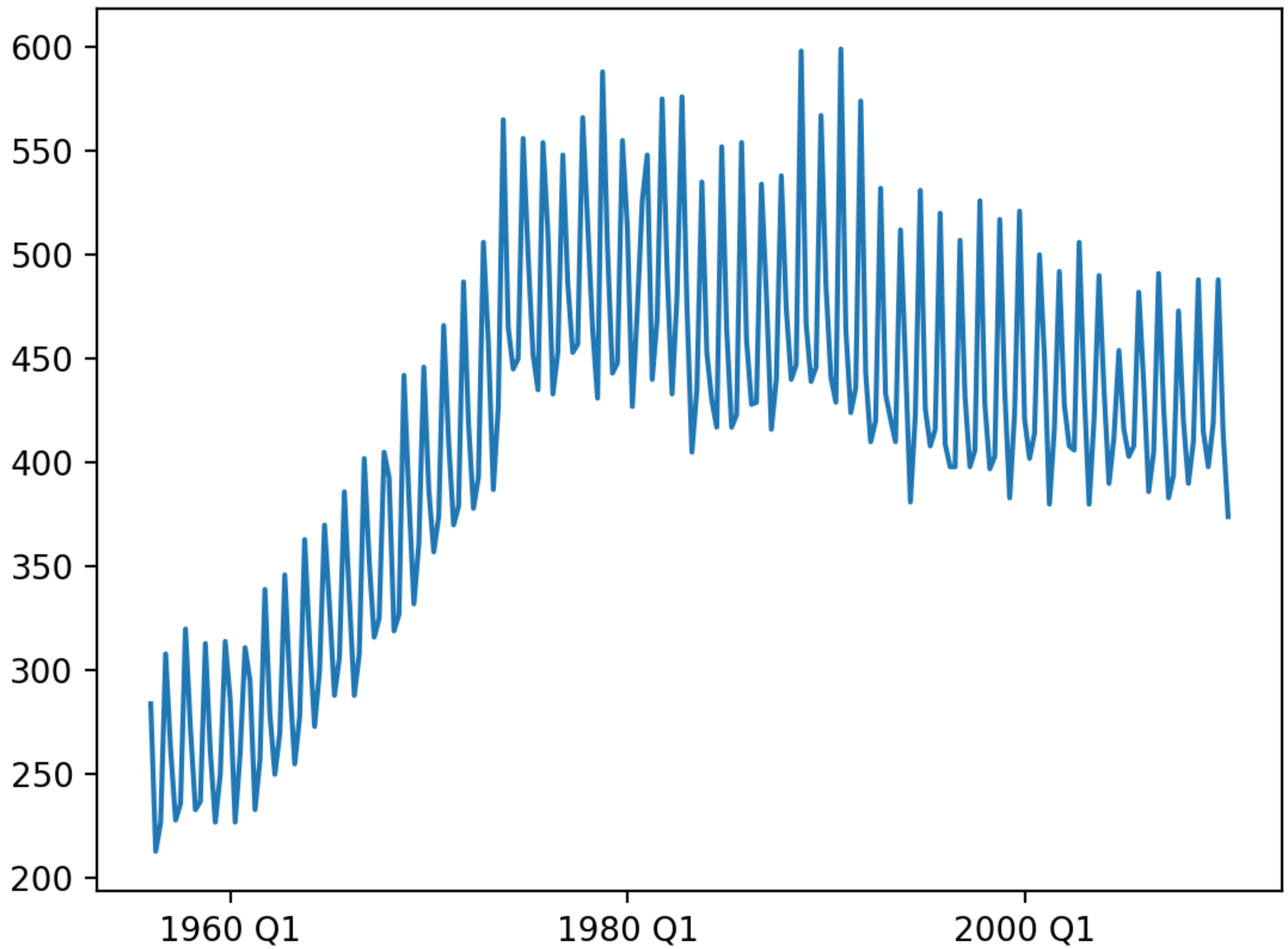
Seasonal Dummy Variables

- Example:
  - Australian beer production (quarterly)



Seasonal Dummy Variables

- Obviously, two different trends, 1955 - 1975 and 1975 -



Seasonal Dummy Variables

- Look at the original data:
  - Need to parse time stamps from two different columns
  - Luckily, parse data is up to it

```
Time,Year,Quarter,Beer.Production
1,1956,Q1,284
2,1956,Q2,213
3,1956,Q3,227
4,1956,Q4,308
5,1957,Q1,262
6,1957,Q2,228
7,1957,Q3,236
8,1957,Q4,320
9,1958,Q1,272
10,1958,Q2,233
```

```
def get_data():
    df_ab = pd.read_csv('AusBeer.csv',
                        sep = ',',
                        parse_dates={'period':['Year', 'Quarter']}
                        )
    df_ab = df_ab.set_index('period')
    return df_ab
```

Seasonal Dummy Variables

- Aside:
  - When we draw the graph, need to select x-ticks

```
def show(df_ab):
    plt.plot(df_ab['Beer.Production'])
    plt.plot(df_ab['pred'])
    plt.xticks(['1960 Q1', '1980 Q1', '2000 Q1'])
    plt.show()
```

Seasonal Dummy Variables

- First, linear regression just on beer production
  - without accounting for the influence of the quarters

```
df_ab = get_data()
df = df_ab.loc['1979 Q1':]
y = df['Beer.Production']
x = df['Time']
model = ols("y~ x",df).fit()
print(model.summary())
```

Seasonal Dummy Variables

- This gives so-so values (as should be expected)

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.169
Model:                            OLS   Adj. R-squared:                  0.163
Method:                 Least Squares   F-statistic:                     25.26
Date:                Fri, 03 Jul 2020   Prob (F-statistic):           1.71e-06
Time:                        23:43:30   Log-Likelihood:                -667.55
No. Observations:                 126   AIC:                             1339.
Df Residuals:                     124   BIC:                             1345.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept    545.7075     19.076     28.606      0.000     507.950     583.465
x             -0.6003      0.119     -5.026      0.000      -0.837      -0.364
==============================================================================
Omnibus:                       13.659   Durbin-Watson:                   2.182
Prob(Omnibus):                  0.001   Jarque-Bera (JB):               15.806
Skew:                           0.858   Prob(JB):                     0.000370
Kurtosis:                       2.740   Cond. No.                         701.
==============================================================================
```

Seasonal Dummy Variables

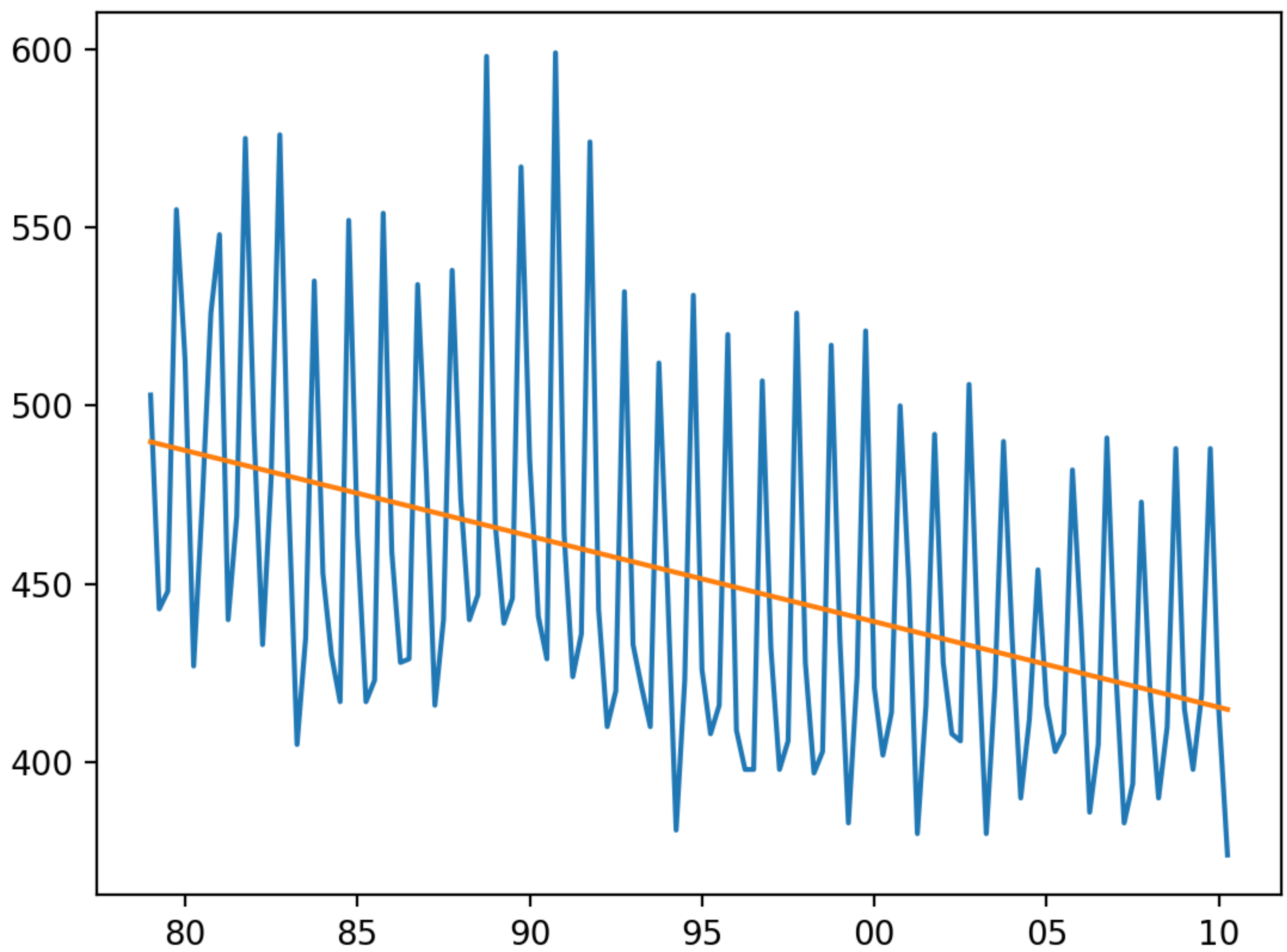
- But already some nice trend
  - Get the model parameters and add a new column with predicted values

```
a, b = model.params
df_ab['pred'] = a + b*df.Time
```

  - Then display

```
df_ab.dropna(inplace=True)
plt.plot(df_ab['Beer.Production'])
plt.plot(df_ab['pred'])
plt.xticks(['1980 Q1', '1985 Q1','1990 Q1',
      '1995 Q1', '2000 Q1', '2005 Q1', '2010 Q1'],
       ['80', '85', '90', '95', '00', '05', '10'])
```

Seasonal Dummy Variables



Seasonal Dummy Variables

- Now, let's try using seasonal dummy variables
  - Create four new columns

```
def transform(df_ab):
    df_ab['s1'] = [ 1 if 'Q1' in df_ab.index[i]
           else 0 for i in range(len(df_ab.index))]
    df_ab['s2'] = [ 1 if 'Q2' in df_ab.index[i]
           else 0 for i in range(len(df_ab.index))]
    df_ab['s3'] = [ 1 if 'Q3' in df_ab.index[i]
           else 0 for i in range(len(df_ab.index))]
    df_ab['s4'] = [ 1 if 'Q4' in df_ab.index[i]
           else 0 for i in range(len(df_ab.index))]
```

Seasonal Dummy Variables

- Create a slice since 1979

```
df_ab = get_data()
transform(df_ab)
df = df_ab.loc['1979 Q1':]
```

Seasonal Dummy Variables

- Set up the model including the seasonal parameters

```
y = df['Beer.Production']
x = df['Time']
x1 = df['s1']
x2 = df['s2']
x3 = df['s3']
x4 = df['s4']
model = ols("y~ x",df).fit()
print(model.summary())
a, b, b1, b2, b3, b4 = model.params
print(a, b, b1, b2, b3, b4)
```

Seasonal Dummy Variables

OLS Regression Results

Dep. Variable:	y	R-squared:	0.887
Model:	OLS	Adj. R-squared:	0.883
Method:	Least Squares	F-statistic:	237.6
Date:	Fri, 03 Jul 2020	Prob (F-statistic):	2.70e-56
Time:	23:56:27	Log-Likelihood:	-541.83
No. Observations:	126	AIC:	1094.
Df Residuals:	121	BIC:	1108.
Df Model:	4		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	437.5337	5.696	76.809	0.000	426.256	448.811
x	-0.6058	0.045	-13.586	0.000	-0.694	-0.517
x1	107.4220	3.124	34.390	0.000	101.238	113.606
x2	65.4965	3.143	20.837	0.000	59.273	71.720
x3	81.4240	3.156	25.804	0.000	75.177	87.671
x4	183.1911	3.175	57.697	0.000	176.905	189.477

Omnibus:	20.235	Durbin-Watson:	1.948
Prob(Omnibus):	0.000	Jarque-Bera (JB):	34.949
Skew:	0.731	Prob(JB):	2.58e-08
Kurtosis:	5.126	Cond. No.	4.40e+17

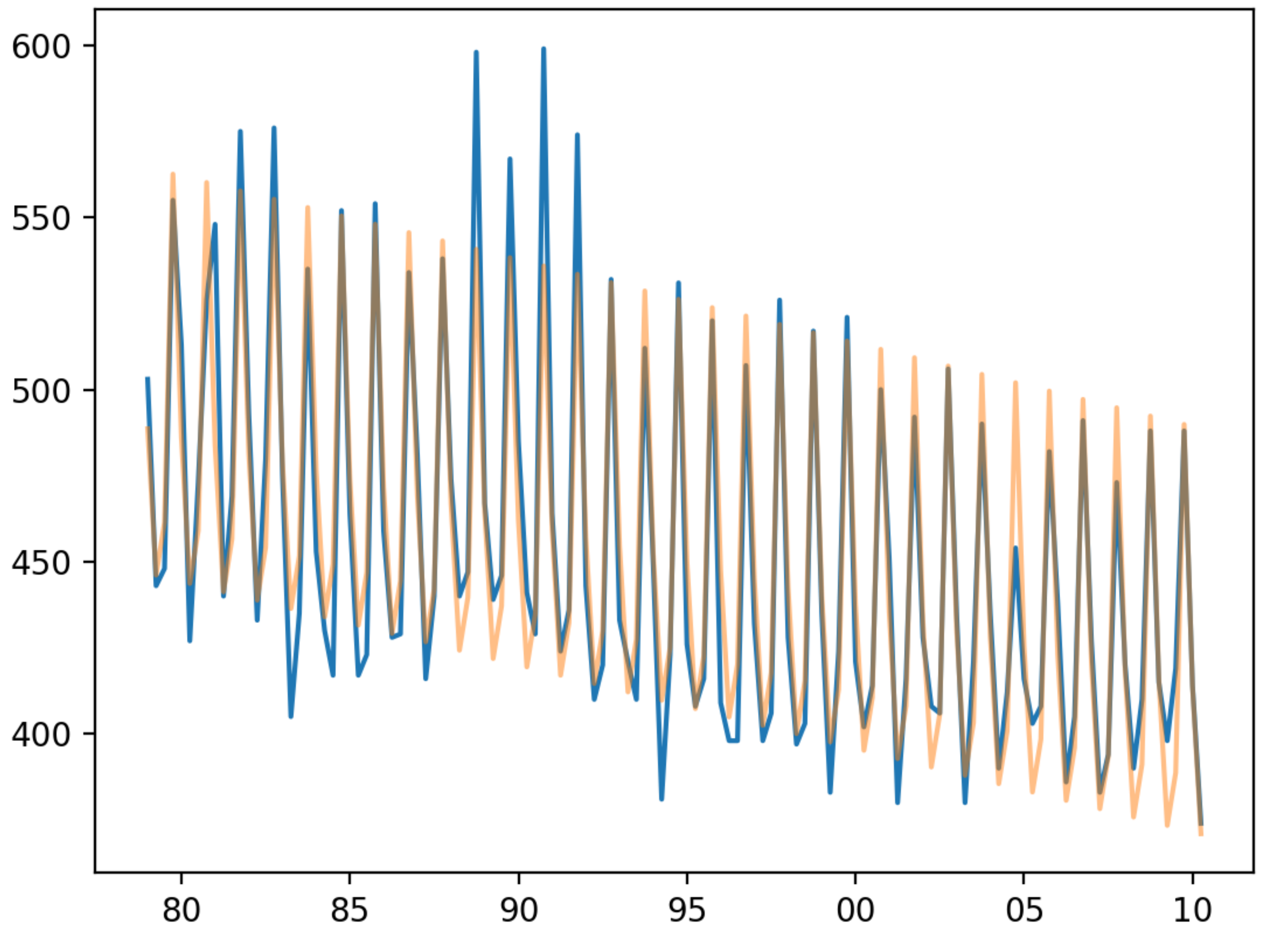
Seasonal Dummy Variables

- Add a new column for the prediction
  - ```
    df_ab['pred'] = a + b*df.Time
    +b1*df.s1+b2*df.s2+b3*df.s3+b4*df.s4
    df_ab.dropna(inplace=True)
    ```
- And plot raw data and prediction values

```
plt.plot(df_ab['Beer.Production'])
plt.plot(df_ab['pred'], alpha = 0.5)

plt.xticks(['1980 Q1', '1985 Q1','1990 Q1',
         '1995 Q1', '2000 Q1', '2005 Q1', '2010 Q1'],
          ['80', '85', '90', '95', '00', '05', '10'])
plt.show()
```
```

Seasonal Dummy Variables



Seasonal Dummy Variables

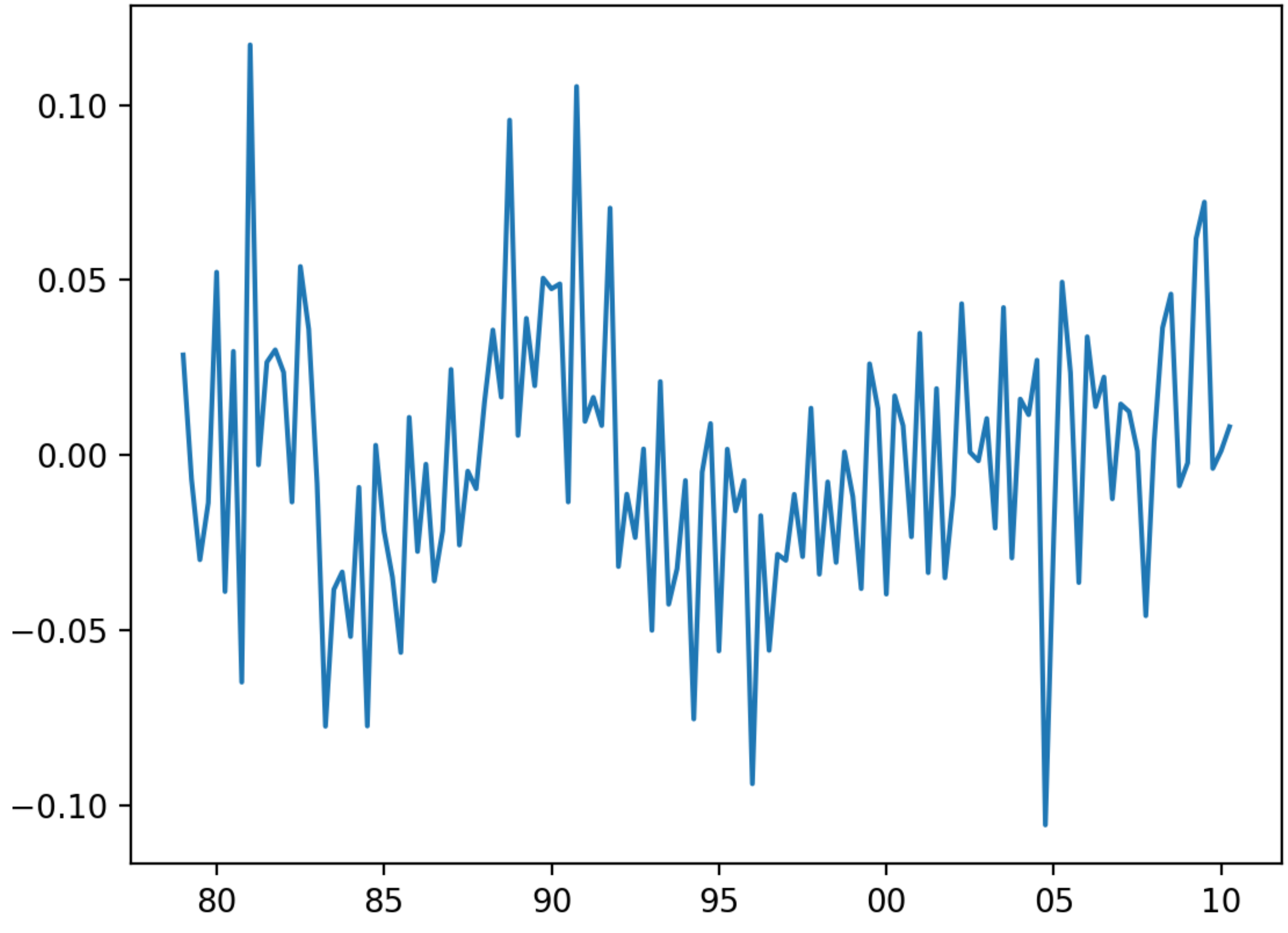
- Why are we not doing better?
  - We only have time and season as explanatory variables
    - Temperature and economy could also explain beer consumption
    - And maybe exports?

Seasonal Dummy Variables

- Let's look at the average error of the prediction
  - Add one more column to the data frame
    - ```
      df_ab['res'] =
      (df_ab['Beer.Production']-
      df_ab['pred'])/df_ab['Beer.Production']
      ```
  - And display
      ```

Seasonal Dummy Variables

- Shows that we are historically with 10% of the linear regression calculated value

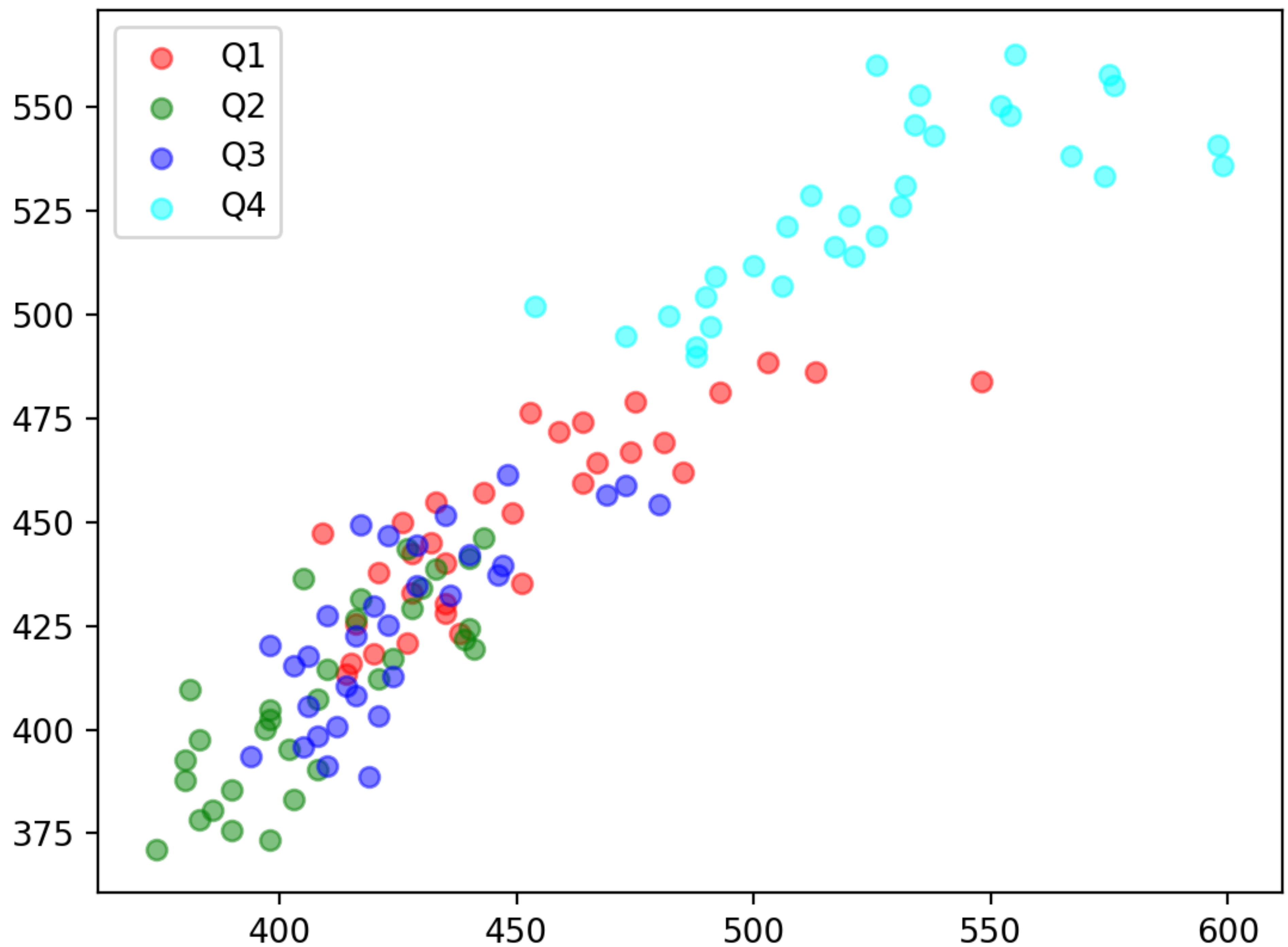


Seasonal Dummy Variables

- We can also look at how well the prediction works

```
plt.scatter(df_ab['Beer.Production'][df_ab['s1']==1],
            df_ab['pred'][df_ab['s1']==1],
            alpha = 0.5, c = 'red', label='Q1')
plt.scatter(df_ab['Beer.Production'][df_ab['s2']==1],
            df_ab['pred'][df_ab['s2']==1],
            alpha = 0.5, c = 'green', label='Q2')
plt.scatter(df_ab['Beer.Production'][df_ab['s3']==1],
            df_ab['pred'][df_ab['s3']==1],
            alpha = 0.5, c = 'blue', label='Q3')
plt.scatter(df_ab['Beer.Production'][df_ab['s4']==1],
            df_ab['pred'][df_ab['s4']==1],
            alpha = 0.5, c = 'cyan', label='Q4')
plt.legend()
plt.show()
```

Seasonal Dummy Variables



Classical Decomposition

- Simple method with periods of *m* on y
  - quarters: *m*=4, business weeks: *m*=5, years: *m* = 365
  - Additive decomposition: $y = T + S + R$
    - Use moving average of size *2m* (or *m*) to estimate T
    - Calculate the "*detrended*" series $d_t = y_t - T_t$
    - Seasonal component is the average over all values for that period
      $S_t = \text{Average}\left(\{\ldots, d_{t-2m}, d_{t-m}, d_t, d_{t+m}, d_{t+2m}, \ldots\}\right)$
    - Residual is what is left

Classical Decomposition

- Problems with classical decomposition:
  - Trend cycle not available for the first or last times without extrapolation
  - Rapid rises and falls are smoothed out too much
  - Assumes seasonal components repeat
    - Counter-example:
      - Used to be that electricity consumption peaked in the winter (heating)
      - Now peaks in the summer (air-conditioning)
  - Really difficult to deal with extra-ordinary events (strikes, weather, catastrophes, …)

Classical Decomposition

- Implemented in statsmodels

```
from statsmodels.tsa.seasonal import seasonal_decompose
```

- Needs a time series `x`
- Needs a model: "additive" (default), "multiplicative
- Can give filtering weights
- Period (if `x` not Pandas with frequency)
- two-sided: method used for filtering
- extrapolate_trend: non-zero value or 'freq' to extrapolate
  - Otherwise, NaN values

Classical Decomposition

- Example: Australian beer production (again)

```
def get_data():
    df_ab = pd.read_csv('AusBeer.csv',
           sep = ',',
           parse_dates={'period':['Year', 'Quarter']}
                    )
    df_ab = df_ab.set_index('period')
    return df_ab
```

Classical Decomposition

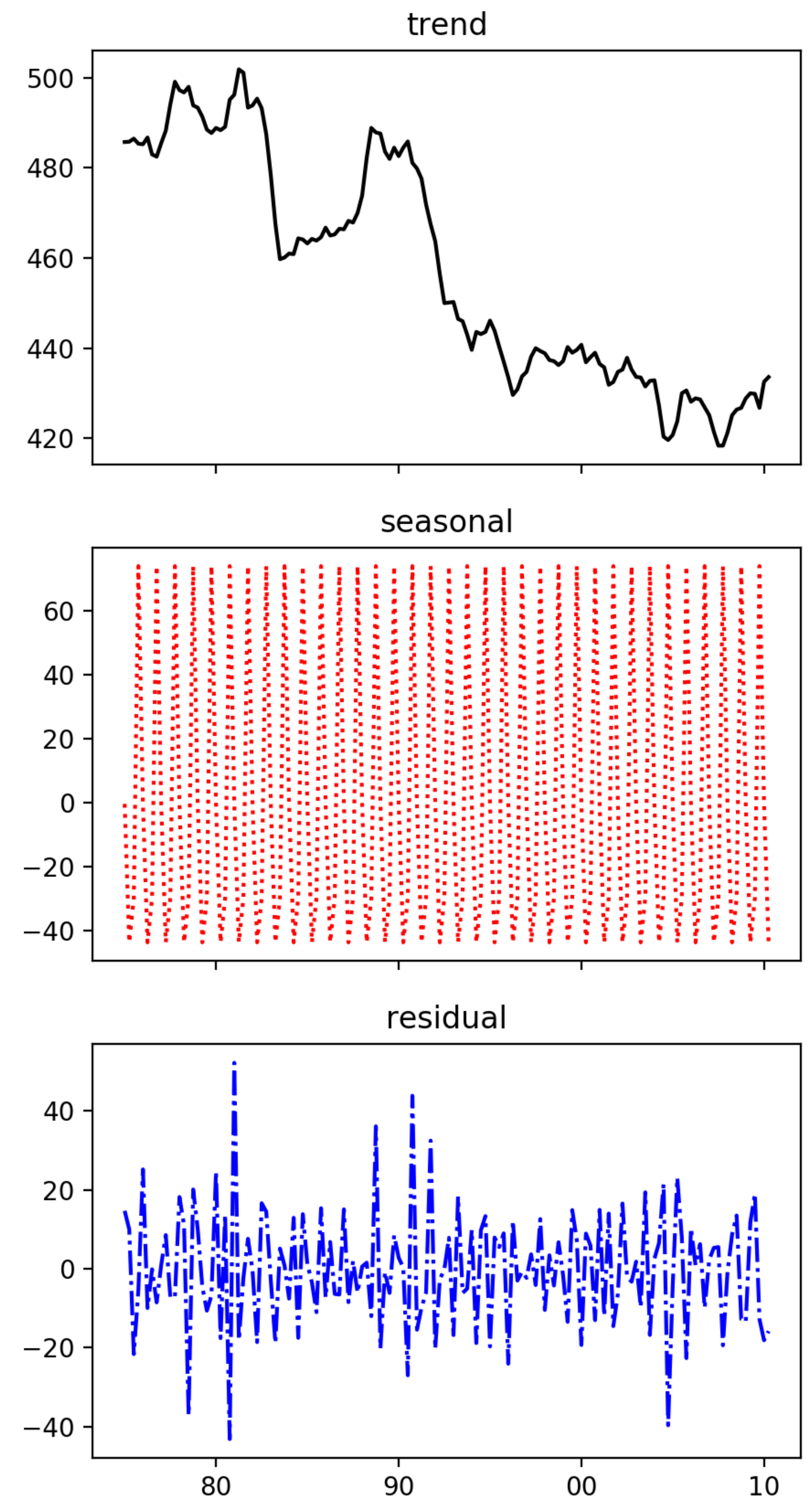
- Limit to values after 1975
  - `df = get_data()['1975 Q1': ]`
- Call decomposition
  - ```
    result = seasonal_decompose(
                 df['Beer.Production'],
                 model = 'additive',
                 period = 4,
                 extrapolate_trend = 3)
    ```
  - result has components `trend, seasonal, resid`
    ```

Classical Decomposition

- Show results:

```
def show(result):
    fig, axs = plt.subplots(3, sharex=True, figsize=(5,10))
    axs[0].plot(result.trend, 'k-')
    axs[1].plot(result.seasonal, 'r:')
    axs[2].plot(result.resid, 'b-.')
    axs[0].set_title('trend')
    axs[1].set_title('seasonal')
    axs[2].set_title('residual')
    axs[2].set_xticks(['1980 Q1', '1990 Q1', '2000 Q1', '2010 Q1'])
    axs[2].set_xticklabels(['80', '90', '00', '10'])
    plt.show()
```

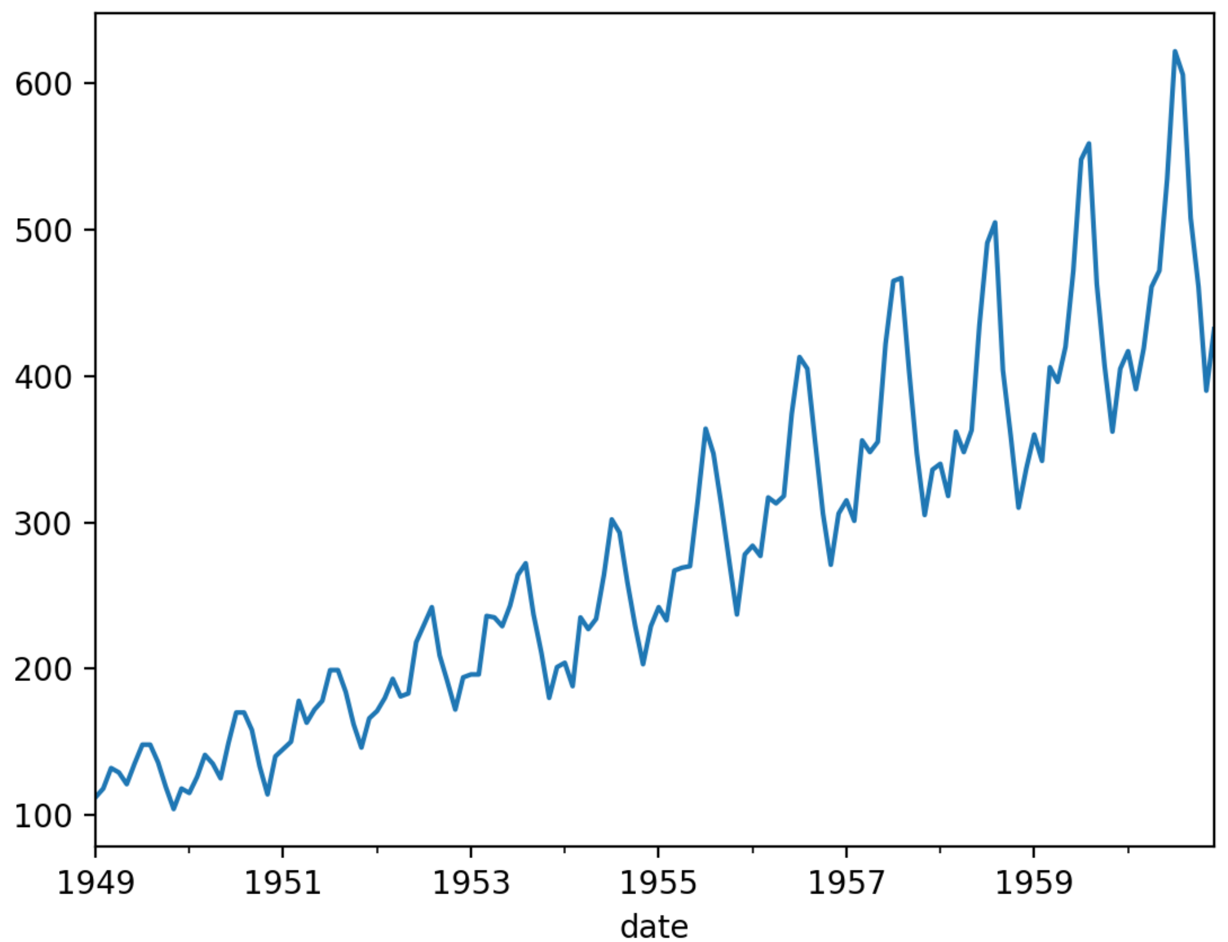
- Can extrapolate the trend
- Add seasonal
- Use residual as a measure of uncertainty

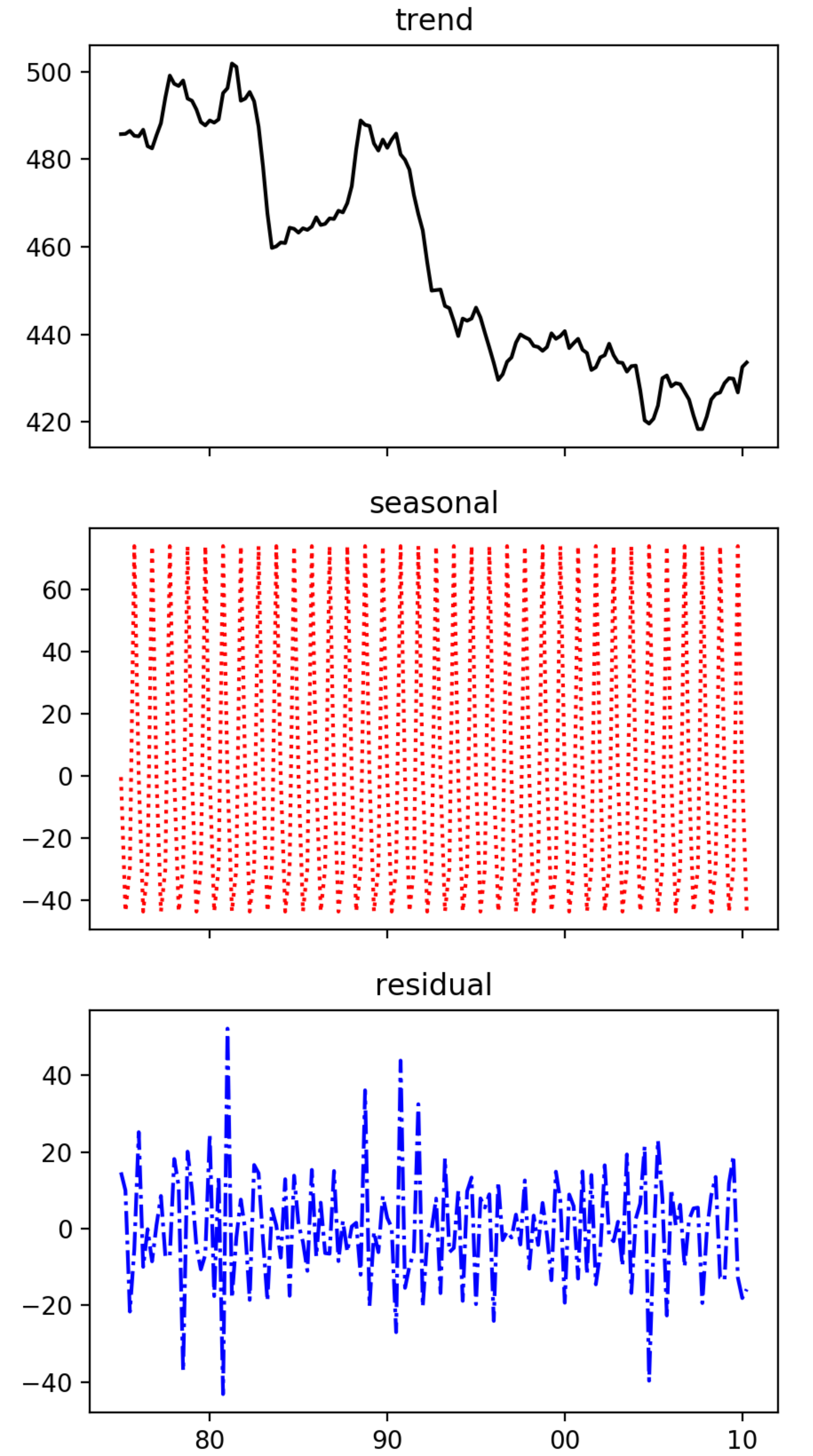


Classical Decomposition

- airline passengers

```
result = seasonal_decompose(df.Passengers,
                            model = 'additive',
                            period = 14,
                            extrapolate_trend = 'freq'
                            )
```


600
500
400
300
200
100
1949
1951
1953
1955
1957
1959
date


trend
500
480
460
440
420
seasonal
60
40
20
0
−20
−40
residual
40
20
0
−20
−40
80
90
00
10

Better Decompositions

- Decomposition has had a 100 year history
    - Better decompositions allow seasonal values to vary
    - **S**easonal **D**ecomposition using **L**OESS (STL)
        - LOESS is based on estimating the trend with a range of functions within a certain window

Better Decompositions

- STL is implemented in statsmodels
  - `from statsmodels.tsa.seasonal import STL`
  - Uses the assumed cyclicity as input (periods)

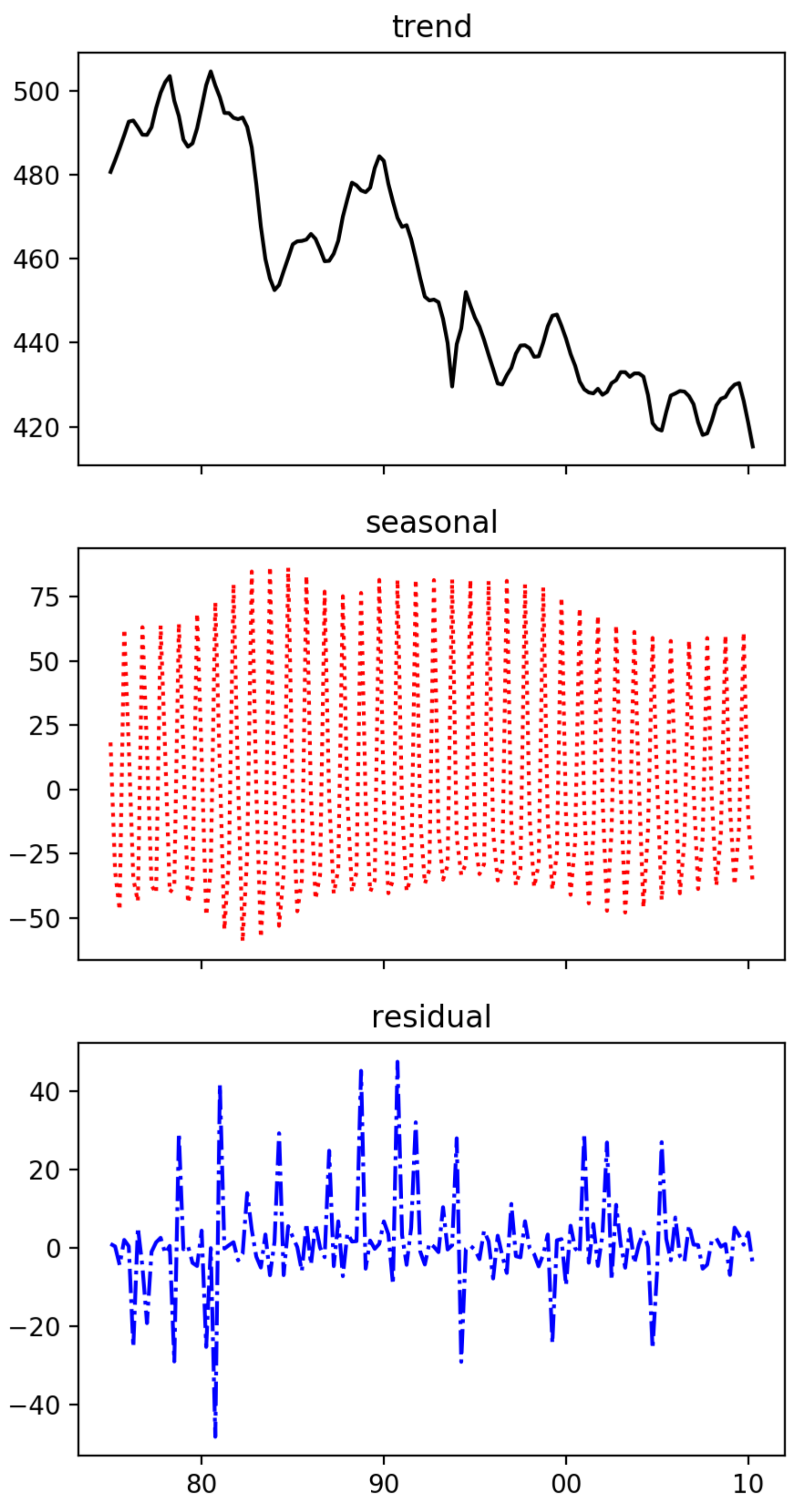
Better Decompositions

- Example: Australian Beer:

```
df = get_data()['1975 Q1': ]
stl = STL(df['Beer.Production'],
             period=4,
             robust=False)
result = stl.fit()
```

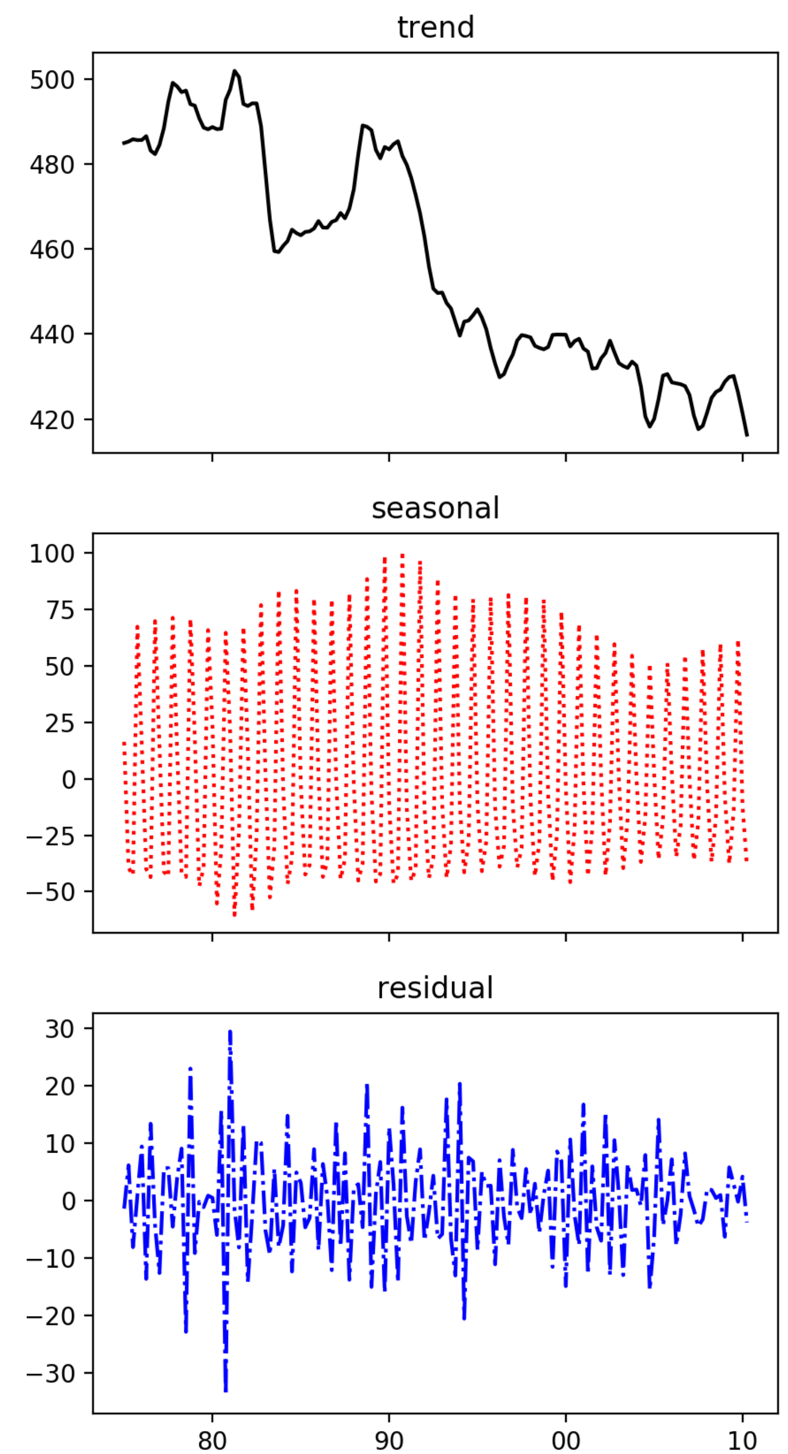
- Result is a triple of trend, seasonal, and resid

Better Decompositions

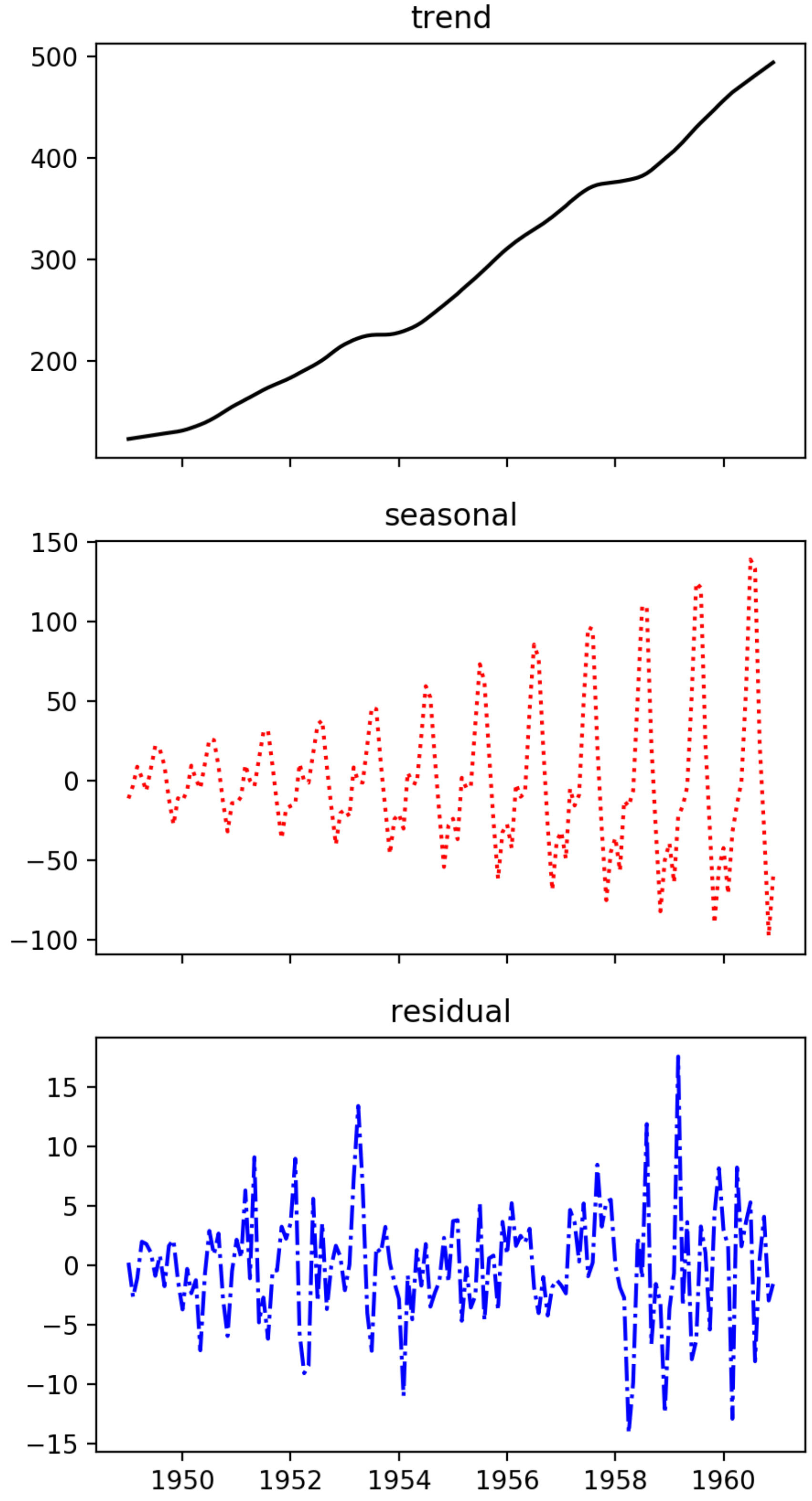


robust

not
robust



Better Decompositions

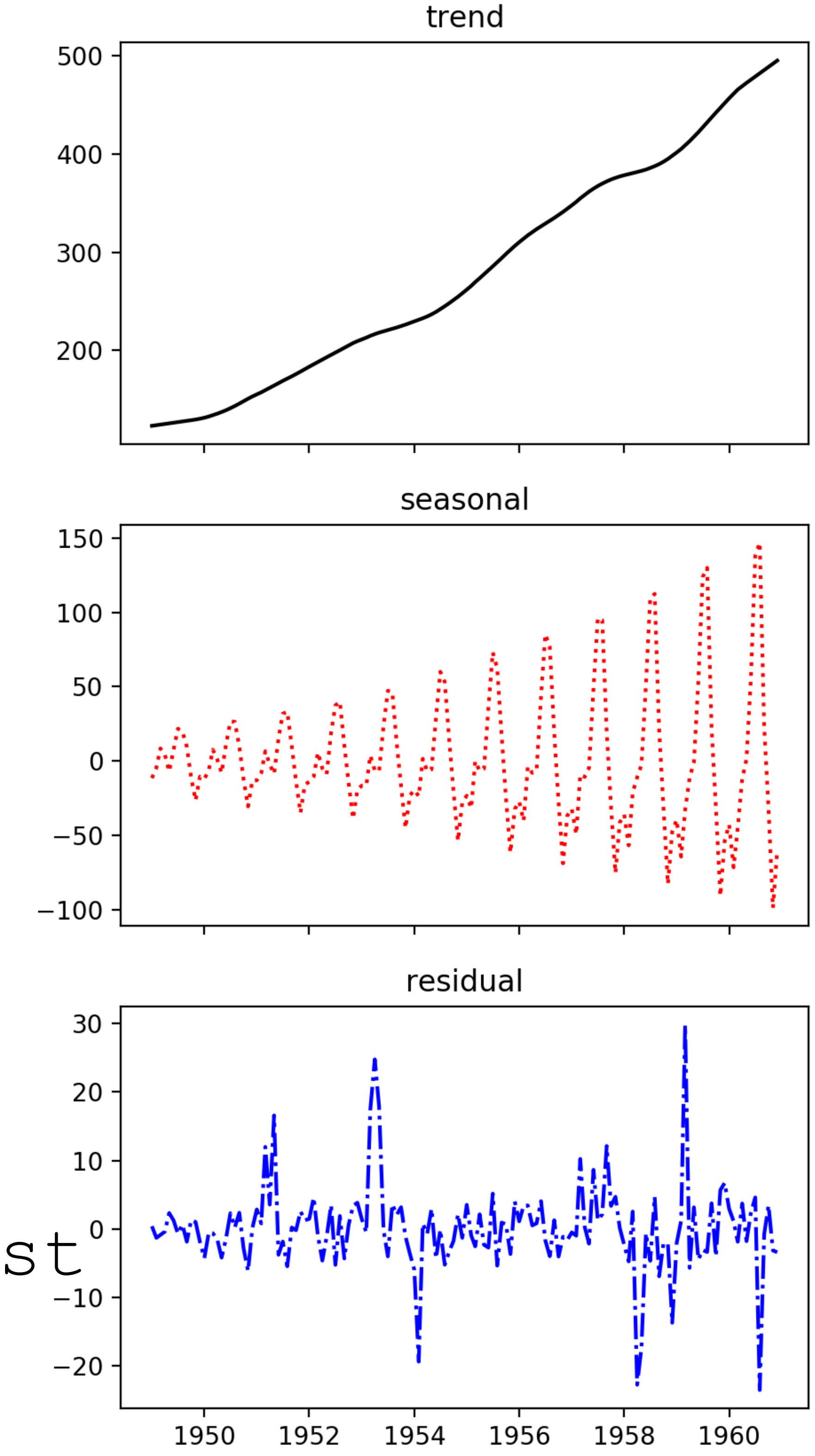


airline

passengers

not
robust

robust



Better Decompositions

- Holt's linear trend method:
  - Simple exponential smoothing for data with trend
    - Estimates series at time t using estimates of the slope obtained as a weighted average
- Holt-Winter's Seasonal Method
  - Adds a seasonal component

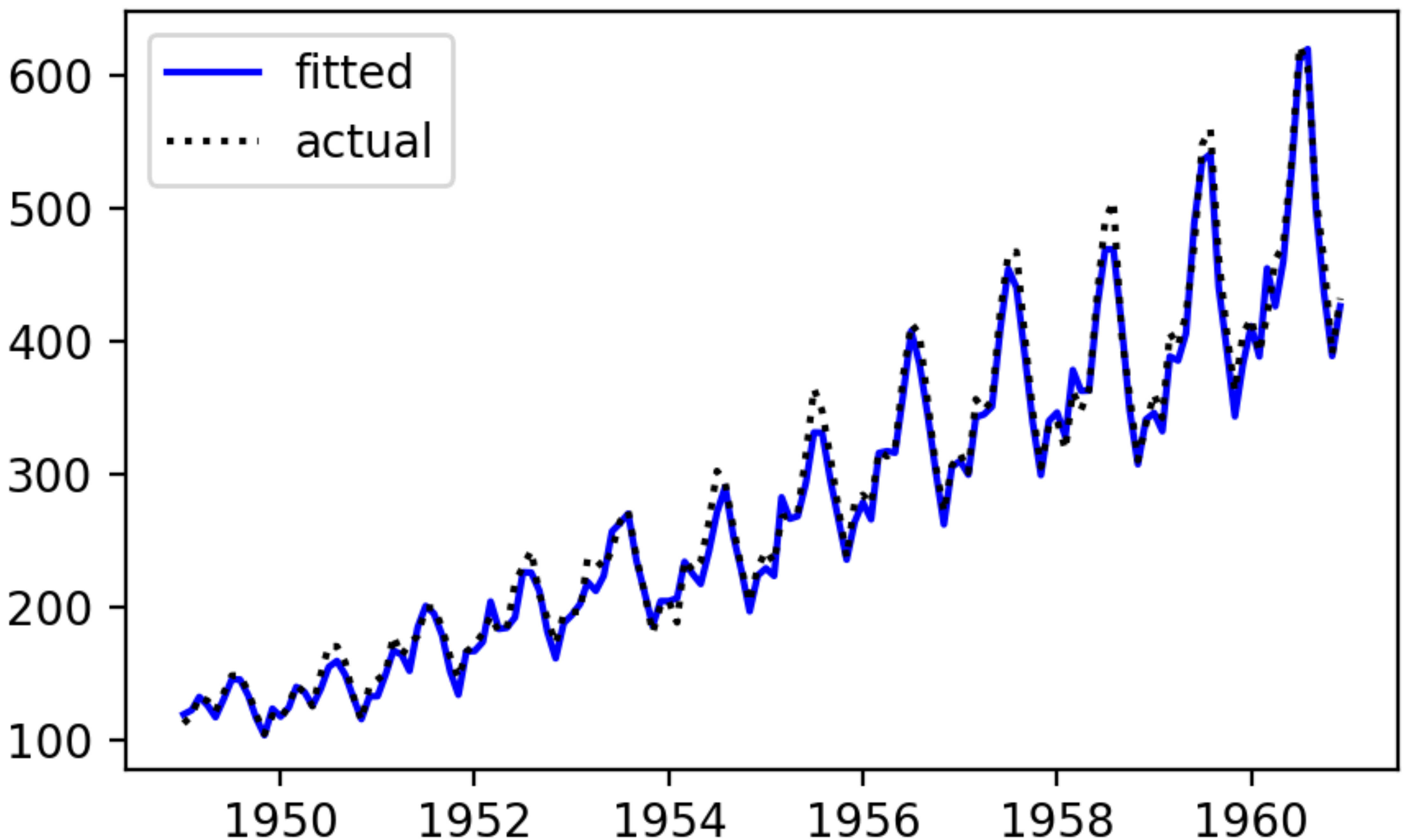
Better Decompositions

- Implemented in statsmodels
  - `from statsmodels.tsa.holtwinters import ExponentialSmoothing as esm`
  - Produces a fitted value (fit) and allows predictions

```
df = get_data()
esm = esm(df['Passengers'],
          seasonal='mul',
          seasonal_periods=12
          )
result = esm.fit()
```

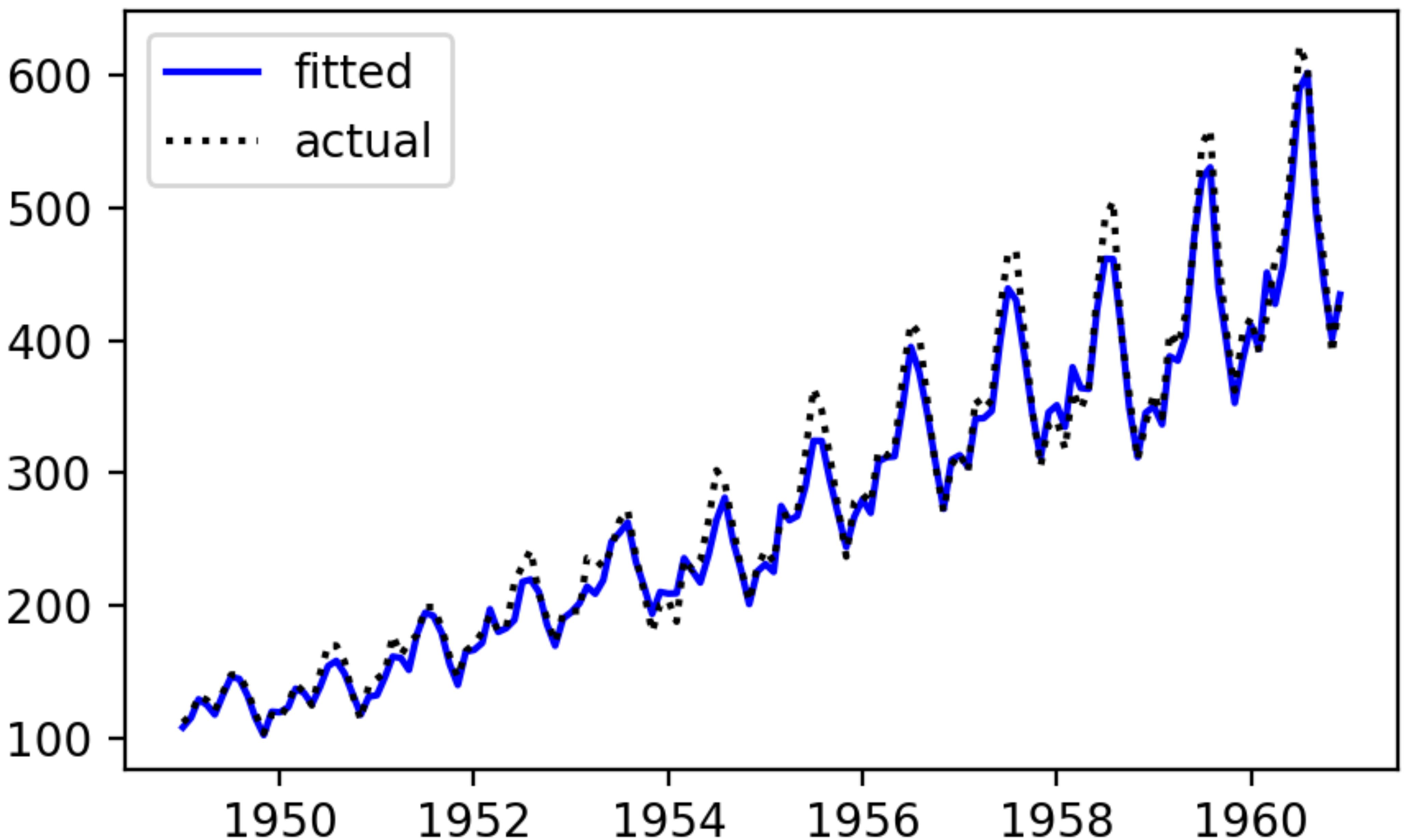
Better Decompositions

- Airline Example (multiplicative fit)



Better Decompositions

- Airline Example additive seasonality

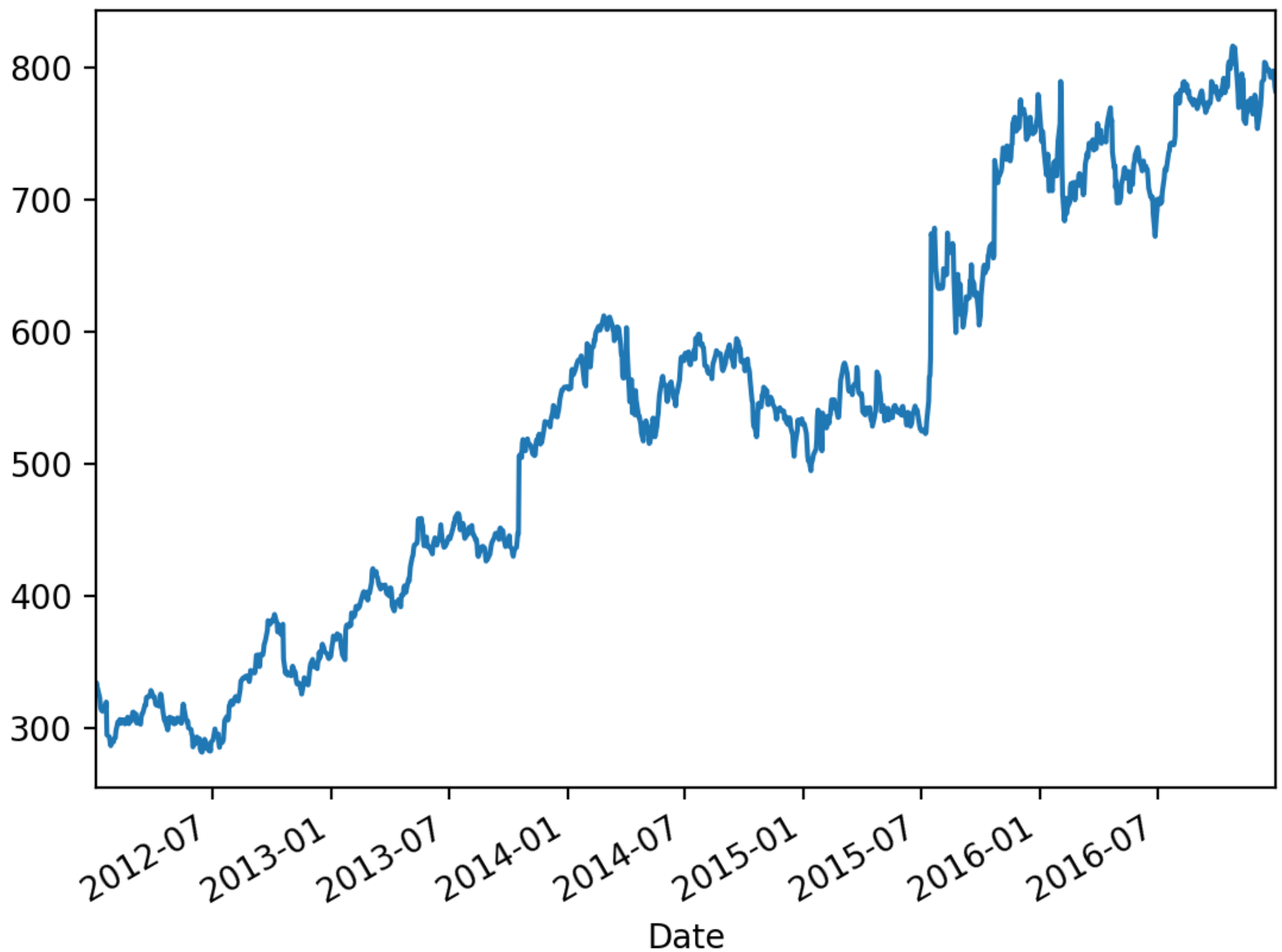


Stationary Time Series

- Properties do not depend on the time at which the series is observed
  - No trend, no seasonality
  - But could be cyclic if cycles have no fixed length

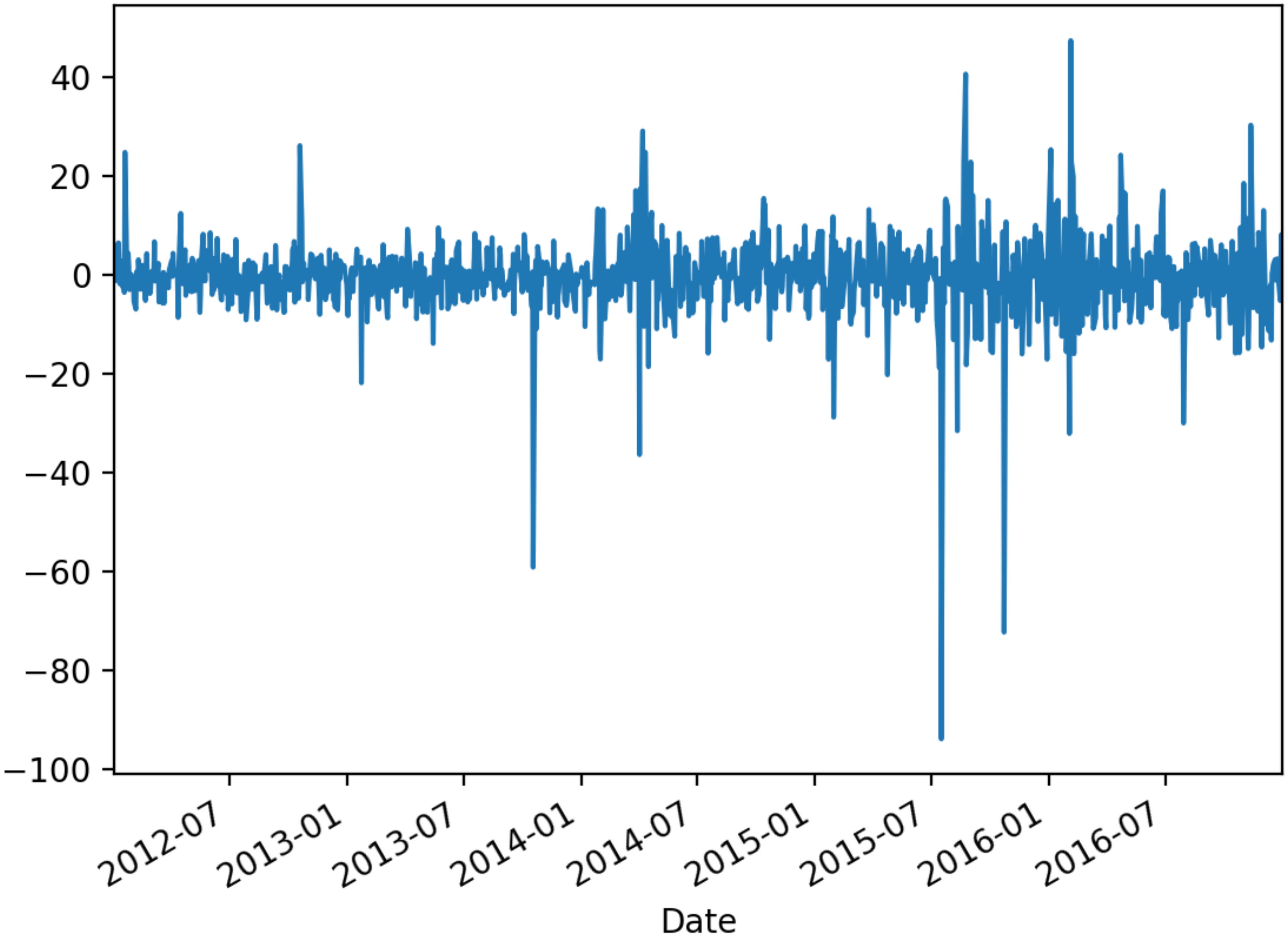
Stationary Time Series

- Example:
  - Google High stock value is not stationary



Stationary Time Series

- But it's daily change is



Stationary Time Series

- Use shift in order to obtain the differences

```
def get_google_data():
    my_df = pd.read_csv('../Pandas/google.csv',
                        parse_dates=[0],
                        index_col=0,
                        converters={
                            'Close': my_converter,
                            'Volume': convert_volume}
                        )
    print(my_df.info())
    #my_df.High.plot()
    (my_df.shift(1).High - my_df.High).plot()
    plt.show()
    return my_df
```

Stationary Time Series

- This is typical
    - **Differencing**
        - Make a non-stationary time series stationary
        - Might have to be repeated several times

Stationary Time Series

- Can look at the Auto-Correlation Function
    - How does the value of a time series relate to the values shifted by *m* periods
- Implemented in Pandas as `autocorr(lag = m)`

Stationary Time Series

- Example: google change

```
my_df = get_google_data()
for i in range(20):
    print(i,
   (my_df.shift(1).High - my_df.High).autocorr(lag=i))
```

```
0 0.9999999999999999
1 0.1208030954108427
2 -0.0353349973850174 35
3 -0.0485249480195700 04
4 -0.0556069387887833 7
5 -0.0064797046657102 605
6 0.0111245377590892 87
7 -0.0390165023147260 3
8 -0.0125207718886038 16
```

Stationary Time Series

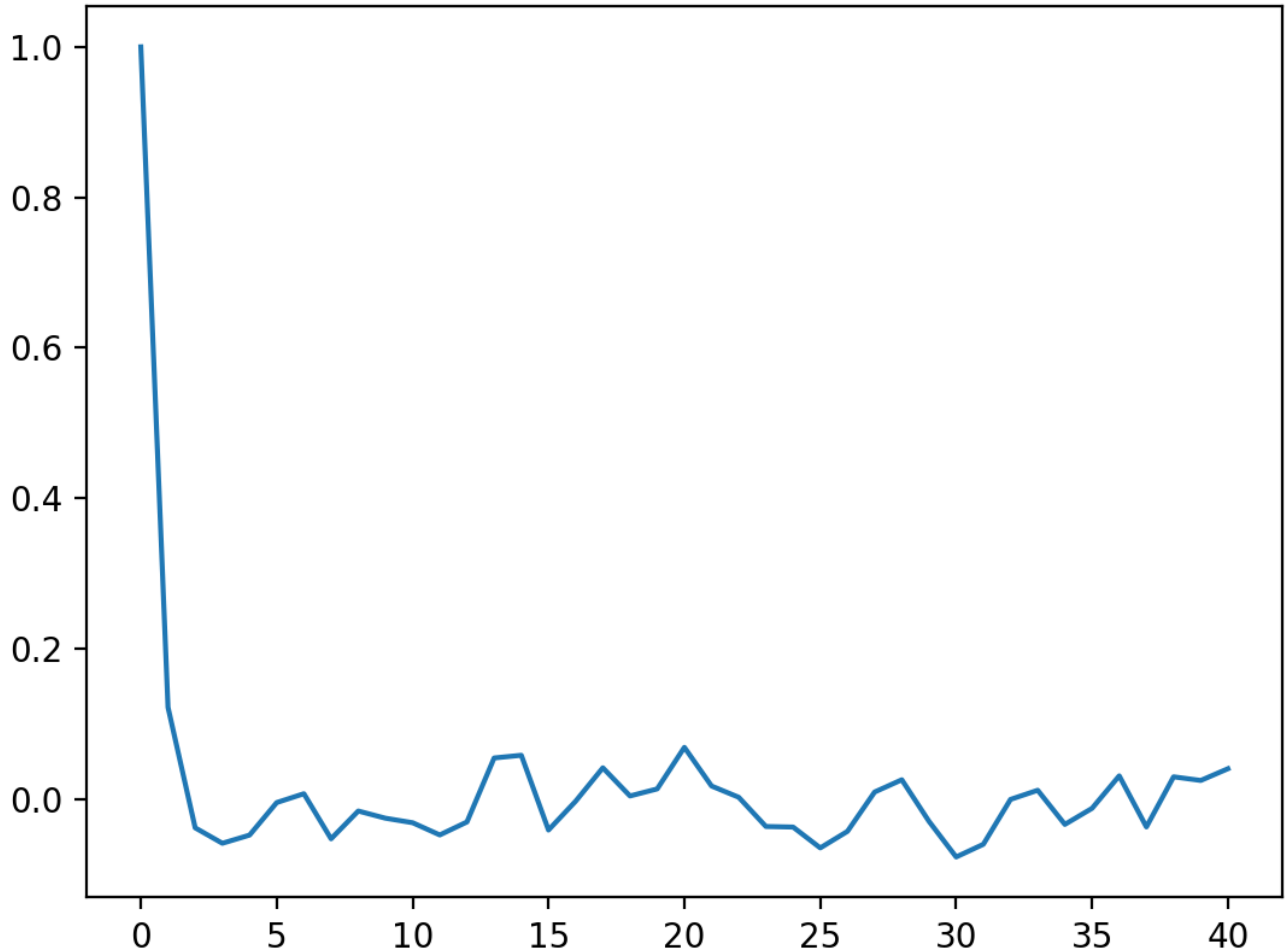
- One version in statsmodels

```
from statsmodels.tsa.stattools import acf, pacf
```

- Calculates ACF as a numpy array

```
my_df = get_google_data()
google = (my_df.shift(1) - my_df).dropna(axis=0)
my_acf = acf(google.High, fft = False)
fig, ax = plt.subplots(1)
ax.plot(my_acf)
```

Stationary Time Series

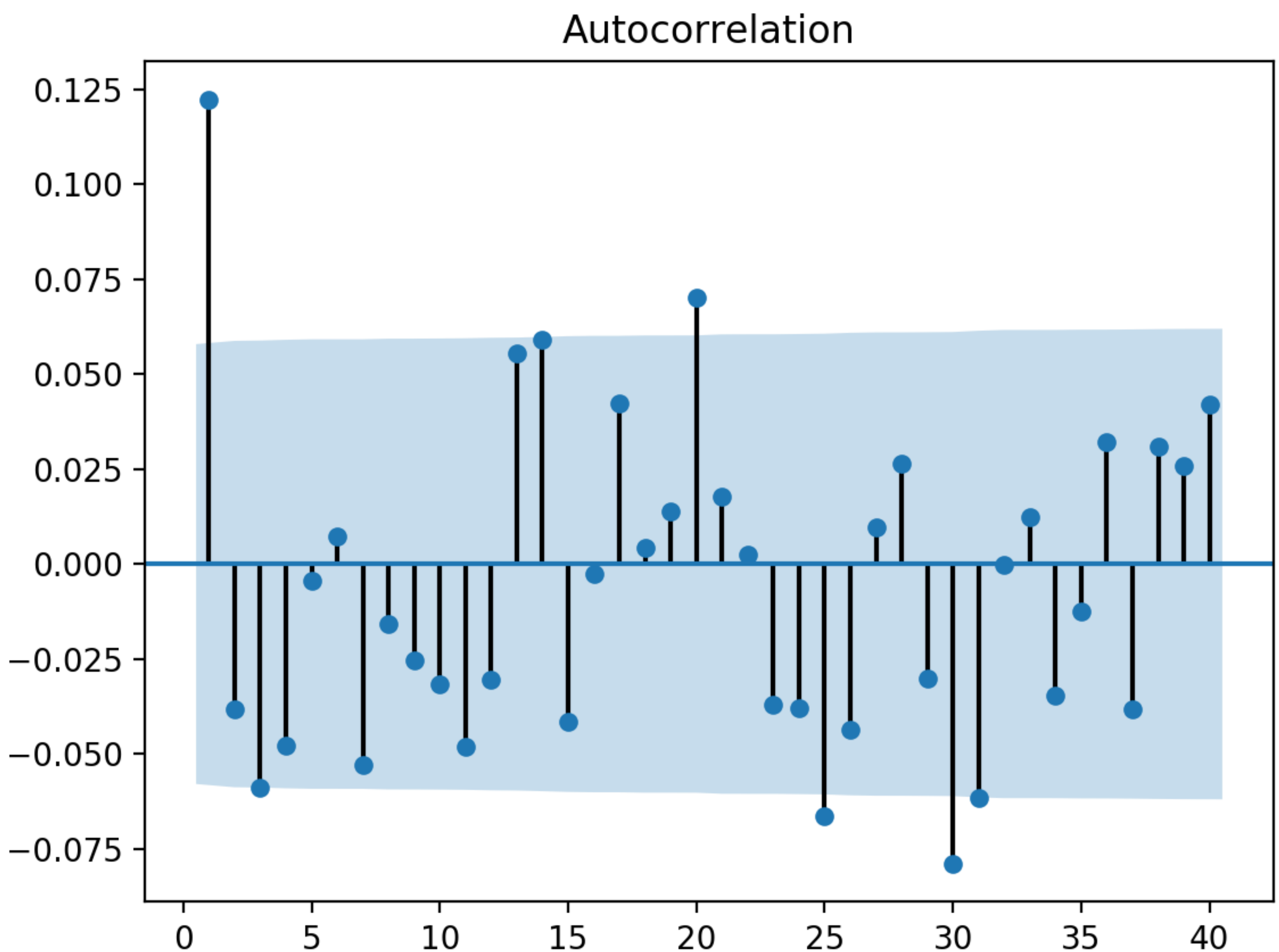


Stationary Time Series

- A graphical version is also available:
  - `import statsmodels.graphics.tsaplots as sgt`

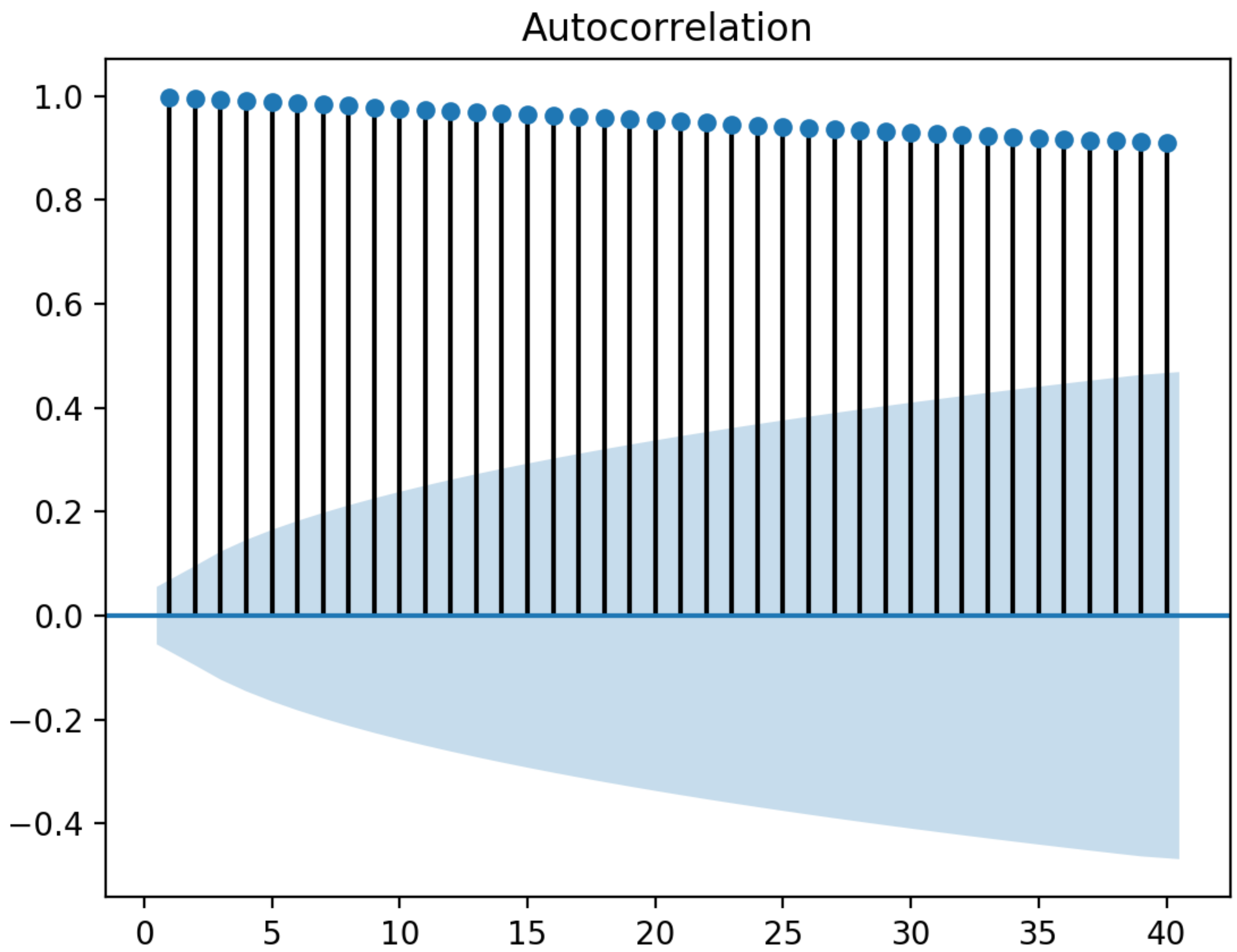
```
sgt.plot_acf(google.High, unbiased = True,
  zero=False, lags = 40)
```

Stationary Time Series



Stationary Time Series

- If we apply the same methodology to the original data
  - Much more autocorrelation

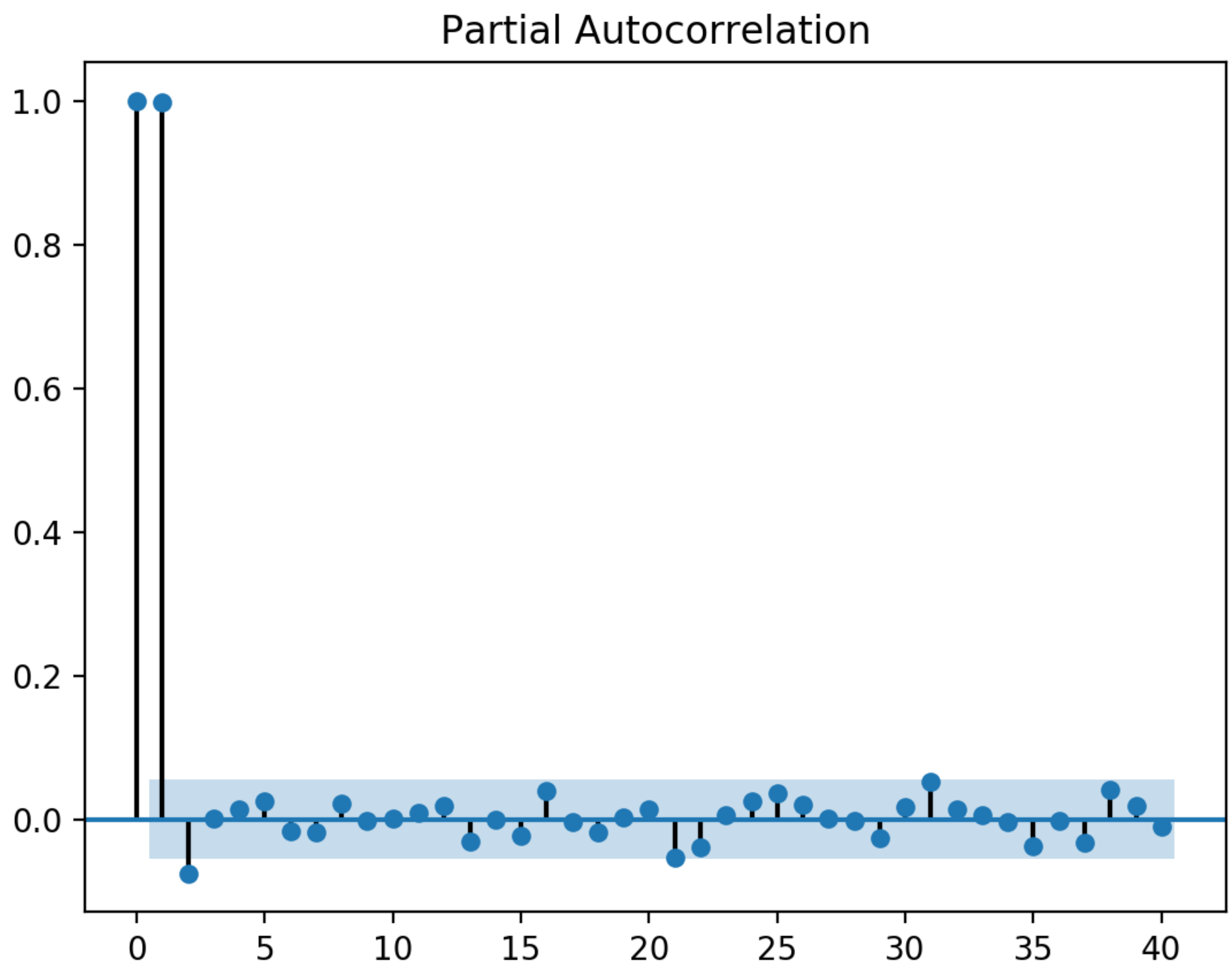


Stationary Time Series

- For lag = 2, the value is mostly because of the correlation between lag = 1
- Use pacm instead
  - Calculates only the auto-correlation not explained by auto-correlation for smaller lags

```
sgt.plot_pacf(my_df.High,  lags = 40)
```

Stationary Time Series



Stationary Time Series

- Autoregression Models
  - Forecast variable of interest using ***linear combination of past values of the variable***
    - $y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_m y_{t-m} + \epsilon_t$
    - With "white error" ϵ_t
    - ***Autoregressive Model of order*** m

Stationary Time Series

- Moving average models
  - Uses past forecast errors
    - $y_t = c + \epsilon_t + \theta_1 \epsilon_{t-1} + \ldots + \theta_q \epsilon_{t-q}$
      - with white noise $\epsilon_t,\ \epsilon_{t-1}, \ldots, \epsilon_{t-q}$

Stationary Time Series

- ARIMA models
  - AutoRegressive Integrated Moving Average

$$y'_t = c + \phi_1 y'_{t-1} + \ldots + \phi_p y'_{t-p} + \theta_1 \epsilon_{t-1} + \ldots + \theta_m \epsilon_{t-q} + \epsilon_t$$

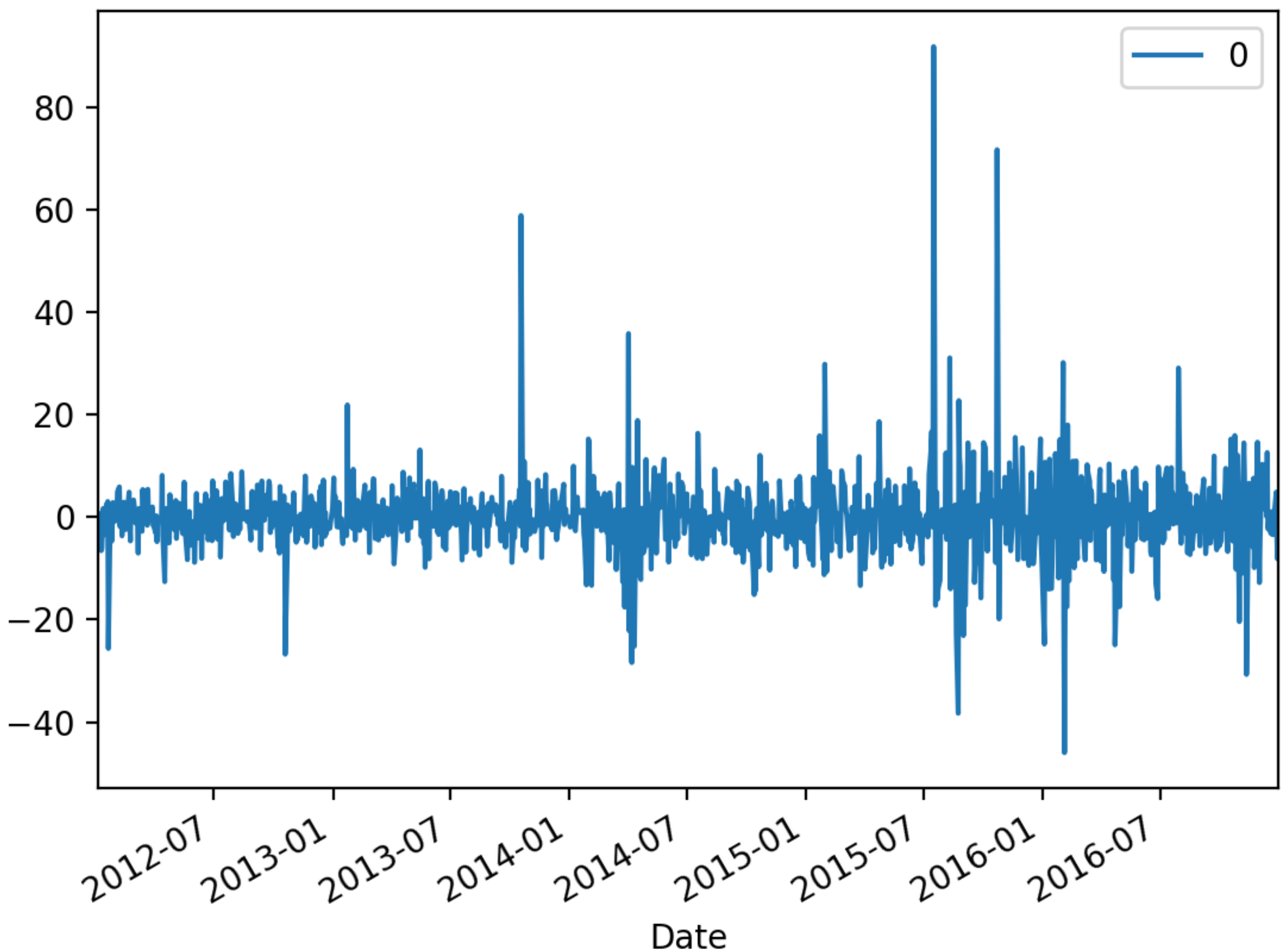
- Differenced value at *t* is
  - autoregressive part of order *p*
  - *d* times differenciated
  - moving average of order *q*
- *ARIMA(p,d,q)*

Stationary Time Series

- Can use statsmodels
  - `from statsmodels.tsa.arima_model import ARIMA`
- Specify degree of ARIMA model and print out its parameters
- Display residual

```
model = ARIMA(my_df.High, order=(1,1,0))
model_fit = model.fit(disp=0)
print(model_fit.summary())
residuals = pd.DataFrame(model_fit.resid)
residuals.plot(label='residual')
plt.show()
```

Stationary Time Series



Seasonal Time Series

- It is possible to extend ARIMA to include a seasonal component
  - In which case we could even put in a trend

```
from statsmodels.tsa.statespace.sarimax import SARIMAX

df = get_data()
my_order = (1,1,1)
my_seasonal_order=(1,1,1,12)
model = SARIMAX(endog = df.Passengers,
                order = my_order,
                seasonal_order = my_seasonal_order)
results = model.fit()
print(results.summary())
results.resid.plot()
plt.show()
```

Stationary Time Series

Stationary Time Series